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Title: On almost everywhere convergence of Fourier series on unbounded Vilenkin groups

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In 1973 GOSSELIN [?] proved that if we have a bounded Vilenkin system, then the Vilenkin–Fourier series of a function in the Lebesgue class L^p for $1 < p$ converges a.e. to the function. It is the most celebrated problem in the harmonic analysis on unbounded Vilenkin groups to give function classes for the elements of which the Fourier series converges almost everywhere. No positive answer is known even for continuous functions in the Lipschitz class. In this paper we give a discretized version of the theorem of Carleson and Hunt, and apply it in order to prove the following theorem with respect to unbounded Vilenkin systems. Let $f \in L^2(G_m)$, and $\sum_{A=0}^{\infty} A^2 \sum_{k=M_A}^{M_{A+1}-1} |\hat{f}(k)|^2 < \infty$. Then we have the a.e. relation $S_n f \rightarrow f$. This immediately implies the a.e. convergence $S_n f \rightarrow f$ for all $f \in \text{Lip}(\alpha, 2)$ ($\alpha > 0$).

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