Year: 2011 | **Vol.: 78** | **Fasc.: 2**

Title: Bilinear character sums over norm groups

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Let k be a finite field with q elements. Let k_n be the extension of k with degree n. Let N_n be the kernel of the norm map $N_{k_n/k} : k_n^\times \to k^\times$. In this paper we estimate the bilinear character sum

$$W_{\rho,\theta}(\psi, \mathcal{U}, \mathcal{V}) = \sum_{U\in\mathcal{U}} \sum_{V\in\mathcal{V}} \rho(U)\theta(V)\psi(UV),$$

where $\mathcal{U}$ and $\mathcal{V}$ are arbitrary subsets of N_n, $\rho(U)$ and $\theta(V)$ are arbitrary bounded complex functions supported on $\mathcal{U}$ and $\mathcal{V}$ and ψ is a nontrivial additive character of k_n. We apply this bound to two problems.

(1) If $\mathcal{S}$, $\mathcal{T}$, $\mathcal{U}$, $\mathcal{V}$ are subsets of N_n, we study the equation $S+T=UV$, where $S \in \mathcal{S}$, $T \in \mathcal{T}$, $U \in \mathcal{U}$, $V \in \mathcal{V}$.

(2) We study the N_n analogy of the sum-product problem.

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