

Title: Some generalizations of the Borsuk–Ulam Theorem

Author(s): Daniel Ventrúscolo , Patricia E. Desideri and Pedro L. Q. Pergher

Let S^n be the n -dimensional sphere, $A : S^n \rightarrow S^n$ the antipodal involution and R^n the n -dimensional euclidean space. The famous Borsuk–Ulam Theorem states that, if $f : S^n \rightarrow R^n$ is any continuous map, then there exists a point $x \in S^n$ such that $f(x) = f(A(x))$. In this paper we discuss some generalizations and variants of this theorem concerning the replacement either of the domain (S^n, A) by other free involution pairs (X, T) , or of the target space R^n by more general topological spaces. For example, we consider the cases where: i) (S^2, A) is replaced by a product involution $(X, T) \times (Y, S) = (X \times Y, T \times S)$, where X and Y are Hausdorff and pathwise connected topological spaces, the involution T is free and the fundamental group of X is a torsion group; ii) R^n is replaced by $M^r \times N^s$, where M^r and N^s are closed manifolds with dimensions r and s , respectively, and $r + s = n$; iii) (S^2, A) is replaced by a product involution as described in i), and R^2 is replaced by the 2-dimensional torus T^2 . We remark that i) includes the case in which $(X, T) \times (Y, S) = (X, T)$, by taking $(Y, S) = (\{\text{point}\}, \text{identity})$, and in particular the popular 2-dimensional Borsuk–Ulam Theorem.

Address:

Daniel Ventrúscolo
Departamento de Matemática
Universidade Federal de São Carlos
Caixa Postal 676
São Carlos, SP 13565-905
Brazil

Address:

Patricia E. Desideri
Departamento de Matemática
Universidade Federal de São Carlos
Caixa Postal 676
São Carlos, SP 13565-905
Brazil

Address:

Pedro L. Q. Pergher
Departamento de Matemática
Universidade Federal de São Carlos
Caixa Postal 676
São Carlos, SP 13565-905
Brazil