

Title: Functional equation of Dhombres type in the real case

Author(s): Ludwig Reich , Jaroslav Smítal and Marta Štefánková

We consider continuous solutions $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+ = (0, \infty)$ of the functional equation $f(xf(x)) = \varphi(f(x))$ where φ is a given continuous map $\mathbb{R}_+ \rightarrow \mathbb{R}_+$. If φ is an increasing homeomorphism the solutions are completely described, if not there are only partial results. In this paper we bring some necessary conditions upon a possible range R_f . In particular, if $\varphi|_{R_f}$ has no periodic points except for fixed points then there are at most two fixed points in R_f , and all possible types of R_f and all possible types of behavior of f can be described. The paper contains techniques which essentially simplify the description of the class of all solutions.

Address:

Ludwig Reich
Institut für Mathematik
Karl-Franzens Universität Graz
A-8010 Graz
Austria

Address:

Jaroslav Smítal
Mathematical Institute
Silesian University
CZ-746 01 Opava
Czech Republic

Address:

Marta Štefánková
Mathematical Institute
Silesian University
CZ-746 01 Opava
Czech Republic