

Title: Minkowski-type inequalities for means generated by two functions and a measure

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Given two continuous functions $f, g : I \rightarrow R$ such that g is positive and f/g is strictly monotone, and a probability measure μ on the Borel subsets of $[0, 1]$, the two variable mean $M_{f,g;\mu} : I^2 \rightarrow I$ is defined by

$$M_{f,g;\mu}(x, y) := \left(\frac{f}{g}\right)^{-1} \left(\frac{\int \int_0^1 f(tx + (1-t)y) d\mu(t)}{\int \int_0^1 g(tx + (1-t)y) d\mu(t)} \right) \quad (x, y \in I).$$

The aim of this paper is to study Minkowski-type inequalities for these means, i.e., to find conditions for the generating functions $f_0, g_0 : I_0 \rightarrow R$, $f_1, g_1 : I_1 \rightarrow R$, ..., $f_n, g_n : I_n \rightarrow R$, and for the measure μ such that

$$M_{f_0,g_0;\mu}(x_1 + \cdots + x_n, y_1 + \cdots + y_n) \underset{[\geq]}{\leq} M_{f_1,g_1;\mu}(x_1, y_1) + \cdots + M_{f_n,g_n;\mu}(x_n, y_n)$$

holds for all $x_1, y_1 \in I_1$, ..., $x_n, y_n \in I_n$ with $x_1 + \cdots + x_n, y_1 + \cdots + y_n \in I_0$. The particular case when the generating functions are power functions, i.e., when the means are generalized Gini means is also investigated.

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