

**Title:** Nilpotency class of symmetric units of group algebras

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Let  $F$  be a field of odd prime characteristic  $p$ ,  $G$  a group,  $U$  the group of units in the group algebra  $FG$ , and  $U^+$  the subgroup of  $U$  generated by the elements of  $U$  fixed by the anti-automorphism of  $FG$  which inverts all elements of  $G$ . It is known that  $U$  is nilpotent if  $G$  is nilpotent and the commutator subgroup  $G'$  has  $p$ -power order, and then the nilpotency class of  $U$  is at most the order of  $G'$ ; this bound is attained if and only if  $G'$  is cyclic and not a Sylow subgroup of  $G$ . Adalbert Bovdi and János Kurdics proved the ‘if’ part of this last statement by exhibiting a nontrivial commutator of the relevant weight. For the special case when  $G$  is a nonabelian torsion group (so  $G'$  cannot possibly be a Sylow subgroup), the present paper identifies such a commutator in  $U^+$ , showing (Theorem 1) that the same bound is attained even by the nilpotency class of this subgroup. We do not know what happens when  $G'$  is not a Sylow subgroup but  $G$  is not torsion. It can happen that  $U^+$  is nilpotent even though  $U$  is not. The torsion groups  $G$  which allow this are known (from the work of Gregory T. Lee) to be precisely the direct products of a finite  $p$ -group  $P$ , a quaternion group  $Q$  of order 8, and an elementary abelian 2-group. Theorem 2: in this case, the nilpotency class of  $U^+$  is strictly smaller than the nilpotency index of the augmentation ideal of the group algebra  $FP$ , and this bound is attained whenever  $P$  is a powerful  $p$ -group. The nonabelian group  $P$  of order 27 and exponent 3 is not powerful, yet the  $G = P \times Q$  formed with this  $P$  also leads to a  $U^+$  attaining the general bound, so here a necessary and sufficient condition remains elusive.

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