

Title: Lattice-like translation ball packings in Nil space

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Nil geometry is one of the eight homogeneous Thurston 3-geometries: $\mathbf{E}^3, \mathbf{S}^3, \mathbf{H}^3, \mathbf{S}^2 \times \mathbf{R}, \mathbf{H}^2 \times \mathbf{R}, \widetilde{\mathbf{SL}_2\mathbf{R}}, \mathbf{Nil}, \mathbf{Sol}$. **Nil** can be derived from W. HEISENBERG's famous real matrix group. The notion of *translation curve* and translation ball can be introduced by initiative of E. MOLNÁR (see [MS], [MSz], [Sz10]). P. SCOTT in [S] defined **Nil** lattices to which *lattice-like translation ball packings* can be defined. In our work we will use the projective model of **Nil** geometry introduced by E. MOLNÁR in [M97]. In this paper we have studied the translation balls of **Nil** space and computed their volume. Moreover, we have proved in Theorems 4.1–4.2 that the density of the optimal lattice-like translation ball packing for every natural lattice parameter $1 \leq k \in \mathbb{N}$ is in interval $(0.7808, 0.7889)$ and if $r \in (0, r_d]$ ($r_d \approx 0.7456$) then the optimal density is $\delta_{\Gamma}^{opt} \approx 0.7808$. Meanwhile we can apply a nice general estimate of L. FEJES TÓTH [LFT] in our Theorem 4.2. From Corollary 4.2 we shall see that the kissing number of the lattice-like ball packings is less than or equal to 14 and the optimal ball packing is realizable in case of equality. We formulate a conjecture for δ_{Γ}^{opt} , where the density of the conjectural densest packing is $\delta_{\Gamma}^{opt} \approx 0.7808$ for lattice parameter $k = 1$, larger than the Euclidean one ($\frac{\pi}{\sqrt{18}} \approx 0.74048$), but less than the density of the densest lattice-like *geodesic* ball packing in **Nil** space known till now [Sz07]. The kissing number of the translation balls in that packing is 14 as well.

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