

Title: Characterization of additive maps ξ -Lie derivable at zero on von Neumann Algebras

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Let $\mathcal{M}$ be any von Neumann algebra with the center $\mathcal{Z}(\mathcal{M})$. For any scalar ξ , denote by $[A, B]_{\xi} = AB - \xi BA$ the ξ -Lie product of $A, B \in \mathcal{M}$. Assume that $L : \mathcal{M} \rightarrow \mathcal{M}$ is an additive map. It is shown that, if $\mathcal{M}$ has no central summands of type I_1 or type I_2 , then L satisfies $L([A, B]) = [L(A), B] + [A, L(B)]$ whenever $[A, B] = 0$ if and only if there exists an element $Z_0 \in \mathcal{Z}(\mathcal{M})$, an additive map $h : \mathcal{M} \rightarrow \mathcal{Z}(\mathcal{M})$ and an additive derivation $\varphi : \mathcal{M} \rightarrow \mathcal{M}$ such that $L(A) = \varphi(A) + h(A) + Z_0 A$ for all $A \in \mathcal{M}$; if $\mathcal{M}$ has no central summands of type I_1 , then L satisfies $L([A, B]_{\xi}) = [L(A), B]_{\xi} + [A, L(B)]_{\xi}$ whenever $[A, B]_{\xi} = 0$ with $\xi \neq 1$ if and only if $L(I) \in \mathcal{Z}(\mathcal{M})$ and there exists an additive derivation $\varphi : \mathcal{M} \rightarrow \mathcal{M}$ such that $\varphi(\xi A) = \xi \varphi(A)$ and $L(A) = \varphi(A) + L(I)A$ for all $A \in \mathcal{M}$. A result in [22] is improved for prime algebra case.

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