

Title: Discrete generalized Wirtinger's inequalities

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Let n, k be fixed natural numbers $1 \leq k \leq n$. We study the following generalized weighted discrete inequalities of Wirtinger type:

$$\alpha_{\pm}^{(i)} \sum_{j=0}^n p_j |x_j|^2 \leq \sum_j {}^{(i)}r_j |x_j \pm x_{j+k}|^2 \leq \beta_{\pm}^{(i)} \sum_{j=0}^n p_j |x_j|^2$$

where $x_0, x_1, \dots, x_n$ are arbitrary complex numbers, $p_0, p_1, \dots, p_n$ and $r_{-k}, \dots, r_0, r_1, \dots, r_n, \dots, r_{n+k}$ are given positive weights, $\alpha_{\pm}^{(i)}, \beta_{\pm}^{(i)}$ are constants and either the $+$ or the $-$ sign has to be taken. $i = 1, 2, 3, 4$ indicates the type of the summation, for example

$$\sum_j {}^{(2)}r_j |x_j \pm x_{j+k}|^2 = \sum_{j=0}^n r_j |x_j \pm x_{j+k}|^2 \quad \text{with } x_{n+1} = \dots = x_{n+k} = 0.$$

Our aim is to find the best constants $\alpha_{\pm}^{(i)}, \beta_{\pm}^{(i)}$.

The weighted versions with positive sign, shift $k = 1$ and $i = 2, 3$ were studied by G. V. MILOVANOVIĆ and I. Ž. MILOVANOVIĆ [9], the unweighted versions were studied by the author [6].

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