

Corrigenda: The geometry of a Randers rotational surface

By RATTANASAK HAMA (Bangkok), PAKKINEE CHITSAKUL (Bangkok)
 and SORIN V. SABAU (Sapporo)

In our paper [1], amongst other topics, we have determined the structure of the cut locus of a point on a Randers rotational surface by deforming the corresponding Riemannian cut locus using the flow of W . However, without being anything wrong with our proofs, we have started with a wrongly positioned Riemannian first conjugate point of q . After correcting this, our Theorem 1.5 and its proof can be simplified as follows.

Theorem 1.5 (p. 476). *Let $(M, F = \alpha + \beta)$ be a rotational Randers von Mangoldt surface of revolution. Then, for any point $q \neq p$, the Finslerian cut locus $\mathcal{C}_q^{(F)}$ of q is the Jordan arc*

$$\mathcal{C}_q^{(F)} = \{\varphi(s, \tau_q(s)) : s \in [c, \infty)\},$$

where $\varphi(c, \tau_q(c))$ is the first conjugate point of q along the twisted meridian $\varphi(s, \tau_q(s))$.

PROOF OF THEOREM 1.5. (p. 498–500) First of all, observe that from our hypothesis we know that the h -cut locus of q is exactly $\tau_q|_{[c, \infty)}$, where $\tau_q(c)$ is the first h -conjugate point of q along τ_q (see Theorem 7.3.1 in [2]).

We divide our proof in two steps.

At the first step, we will establish the correspondence of h -conjugate points of q along τ_q with the F -conjugate points of q along an F -geodesic from q .

Let $\tilde{x} = \tau_q(c)$ the first h -conjugate point of q along τ_q . Observe that in the case of the Riemannian surface of revolution (M, h) , we must have $c > \rho$, because p is the unique pole for h . This is equivalent to saying that $\tilde{x}$ is conjugate to q along τ_q (see [2], [3]).

Recall that $\tilde{x} = \tau_q(c)$ is the first h -conjugate point of q along τ_q means that the Jacobi field along τ_q given by

$$Y_q(s) = \mathcal{M}_{a_1, \rho}(s) \frac{\partial}{\partial \theta} \Big|_{\tau_q}, \quad s \in [\rho, \infty),$$

where $\mathcal{M}_{a_1, \rho}(s)$ is a smooth function along $\tau_q|_{[\rho, \infty)}$ depending on a constant a_1 chosen such that m' is positive on $[0, a_1]$ and ρ .

Moreover, if consider the vector field $J(s)$, along the twisted meridian $\mathcal{R}_q : [\rho, \infty) \rightarrow M$, $\mathcal{R}_q(s) = \varphi(s, \tau_q(s))$, defined by

$$J(s) := \varphi_{\tau_q, *} (Y_q(s)),$$

then one can see that J is actually a Jacobi field along $\mathcal{R}_q$. Indeed, one can easily verify that the flow φ of W maps the solutions of the Jacobi equation for Y_q into the solutions of the Jacobi equation for $J(s)$, and therefore we have proved that the first F -conjugate point of q is obtained at the intersection of the parallel through the first h -conjugate point with τ_q .

At the second step, we will do the same thing for cut points of q , i.e. we will establish the correspondence of h -cut points of q with the F -cut points of q . Namely, we will show that a point $\tilde{y} \in \tau_q|_{[c, \infty)}$ is an h -cut point of q if and only if the point y , found at the intersection of the parallel through $\tilde{y}$ with the twisted meridian $\{\varphi(s, \tau_q(s)) : s \in [c, \infty)\}$ is an F -cut point of q .

Indeed, such a $\tilde{y}$ is an h -cut point of q if and only if there exists two h -geodesic segments α_1 and α_2 on M from q to $\tilde{y}$ of equal h -length. By making use of Theorem 1.1 and an argument similar to Proposition 3, we can see that under the action of the flow φ the end point $\tilde{y}$ is clearly mapped into the point y described above and the h -maximal geodesic segments α_1 and α_2 are deviated into two F -geodesic segments of same F -length from q to y . This concludes the proof (see Figure 1).

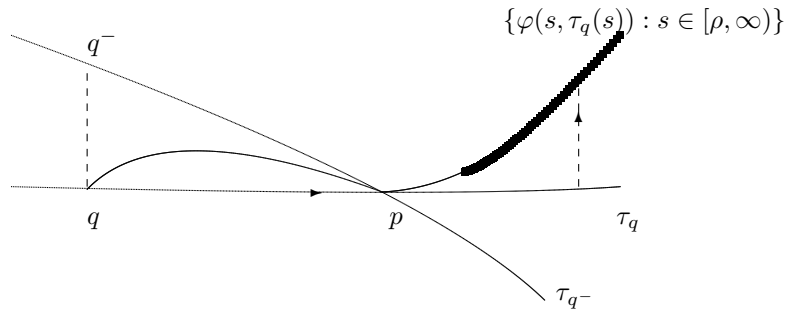


Figure 1. The thick line is the F -cut locus of q .

Here are other two small typos on p. 481, line 2 from bottom. There is a “2” missing in both formulas. The correct formulas are:

$$(L)_F^+(r_0) = \frac{2\pi m(r_0)}{1 + \mu \cdot m(r_0)} \quad \text{and} \quad (L)_F^-(r_0) = \frac{2\pi m(r_0)}{1 - \mu \cdot m(r_0)}.$$

On p. 486, line 10 from top, the formula

$$\frac{\partial \alpha^2}{\partial y^2} = 2\alpha_{22}y^2 \quad \text{should be} \quad \frac{\partial \alpha^2}{\partial y^2} = 2a_{22}y^2.$$

References

- [1] R. HAMA, P. CHITSAKUL and S. V. SABAU, The geometry of a Randers rotational surface, *Publ. Math. Debrecen* **87** (2015), 473–502.
- [2] K. SHIOHAMA, T. SHIOYA and M. TANAKA, The Geometry of Total Curvature on Complete Open Surfaces, Cambridge Tracts in Mathematics **159**, *Cambridge University Press, Cambridge*, 2003.
- [3] M. TANAKA, On the cut loci of a von Mangoldt’s surface of revolution, *J. Math. Soc. Japan* **44** (1992), 631–641.

RATTANASAK HAMA
DEPARTMENT OF APPLIED MATHEMATICS
FACULTY OF SCIENCE
KING MONGKUT’S INSTITUTE
OF TECHNOLOGY LADKRABANG
CHALONGKRUNG RD.
LADKRABANG, BANGKOK, 10520
THAILAND
E-mail: rattanasakhama@gmail.com

PAKKINEE CHITSAKUL
DEPARTMENT OF APPLIED MATHEMATICS
FACULTY OF SCIENCE
KING MONGKUT’S INSTITUTE
OF TECHNOLOGY LADKRABANG
CHALONGKRUNG RD.
LADKRABANG, BANGKOK, 10520
THAILAND
E-mail: kcpakkin@kmitl.ac.th

SORIN V. SABAU
SCHOOL OF SCIENCE
DEPARTMENT OF MATHEMATICS
LIBERAL ARTS EDUCATION CENTER
SAPPORO CAMPUS
TOKAI UNIVERSITY
SAPPORO CAMPUS
5-1-1-1 MINAMISAWA
MINAMIKU, SAPPORO, 005-8601
JAPAN
E-mail: sorin@tokai.ac.jp

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