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Dynamics in a two-species competitive model of plankton allelopathy with delays and feedback controls

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Abstract. In this paper, we propose and investigate a discrete competitive model with delays and feedback controls. With the help of the difference inequality theory, we establish some sufficient conditions which guarantee the permanence of the model. Under some suitable conditions, we show that the periodic solution of the system is global stable. Two example with their numerical simulations are given which are in a good agreement with our theoretical analysis. Our results are new and complement previously known results.

1. Introduction

In a natural ecosystem, the fundamental features of population interactions, such as permanence and competition have been elucidated by empirical and theoretical investigations of the dynamics between two species [1]. Permanence and global attractivity are two important concepts to describe the coexistence of species. In recent few decades, the permanence and global attractivity of various competitive systems have been studied by many scholars. For example, Balbus [2] analyzed the attractivity and stability in the competitive systems

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of PDEs of Kolmogorov type, Hou [3] addressed the permanence of competitive Lotka–Volterra systems with delays, [4] discussed the asymptotic behavior of a stochastic nonautonomous Lotka–Volterra competitive system with impulsive perturbations, Shi *et al.* [5] made a theoretical discussion on the extinction of a nonautonomous Lotka–Volterra competitive system with infinite delay and feedback controls, Liu and Wang [4], and Kulenović and Nurkanović [6] focused on the global behavior of a two-dimensional competitive system of difference equations with stocking. For more related work on the permanence and global attractivity behavior of predator-prey models, one can see [7]–[28].

It is well known that the traditional Lotka–Volterra two-species competitive system takes the form

$$\begin{cases} \dot{x}_1(t) = x_1(t)\left[K_1 - \alpha_1 x_1(t) - \beta_{12} x_2(t)\right], \\ \dot{x}_2(t) = x_2(t)\left[K_2 - \alpha_2 x_2(t) - \beta_{21} x_1(t)\right], \end{cases} \tag{1.1}$$

where $x_1(t)$ and $x_2(t)$ denote the population densities (number of cells per liter) of two competing species; K_1, K_2 are the rates of cell proliferation per hour; α_1, α_2 are the rate of intra-specific competition of first and second species, respectively; β_{12}, β_{21} are the rate of inter-specific competition of first and second species, respectively, and $K_1/\alpha_1, K_2/\alpha_2$ are environmental carrying capacities (representing number of cells per liter). The units of $\alpha_1, \alpha_2, \beta_{12}$ and β_{21} are per hour per cell and the unit of time is hours. Considering that each species produces a substance toxic to the other, but only when the other is present, Maynard [29] and Chattopadhyay [30] modified the system (1.1) as the following form

$$\begin{cases} \dot{x}_1(t) = x_1(t)\left[K_1 - \alpha_1 x_1(t) - \beta_{12} x_2(t) - \gamma_1 x_1(t) x_2(t)\right], \\ \dot{x}_2(t) = x_2(t)\left[K_2 - \alpha_2 x_2(t) - \beta_{21} x_1(t) - \gamma_2 x_1(t) x_2(t)\right], \end{cases} \tag{1.2}$$

where γ_1 and γ_2 are the rates of toxic inhibition of the first species by the second and vice versa, respectively, and $\alpha_1, \alpha_2, \beta_{12}, \beta_{21}, \gamma_1$ and γ_2 are positive constants.

Many authors [31]–[42] have argued that discrete time models governed by difference equations are more appropriate to describe the dynamics relationship among populations than continuous ones when the populations have non-overlapping generations. Moreover, discrete time models can also provide efficient models of continuous ones for numerical simulations. In 2014, Wu and Zhang [43] applied the forward Euler scheme to the system (1.2) and obtained the two-species competitive discrete-time system of plankton allelopathy as follows

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \to \begin{bmatrix} x_1 + \delta x_1\left(K_1 - \alpha_1 x_1 - \beta_{12} x_2 - \gamma_1 x_1 x_2\right) \\ x_2 + \delta x_2\left(K_2 - \alpha_2 x_2 - \beta_{21} x_1 - \gamma_2 x_1 x_2\right) \end{bmatrix}. \tag{1.3}$$

By using the center manifold theorem and bifurcation theory, WU and ZHANG [43] investigated the flip bifurcation of system (1.3). Moreover, numerical simulations display interesting dynamical behaviors (including period-doubling orbits and chaotic sets) for the system (1.3).

Considering that the coefficients, in the real world, are not unchanged constants owing to the variation of environment, and the effect of a varying environment is important for evolutionary theory as the selective forces on systems in such a fluctuating environment differ from those in a stable environment, we can modify system (1.2) as the form

$$\begin{cases} \dot{x}_1(t) = x_1(t)\left[K_1(t) - \alpha_1(t)x_1(t) - \beta_{12}(t)x_2(t) - \gamma_1(t)x_1(t)x_2(t)\right], \\ \dot{x}_2(t) = x_2(t)\left[K_2(t) - \alpha_2(t)x_2(t) - \beta_{21}(t)x_1(t) - \gamma_2(t)x_1(t)x_2(t)\right], \end{cases} \tag{1.4}$$

where the coefficients $K_i(t), \alpha_i(t), \gamma_i(t)(i = 1, , 2), \beta_{12}(t), \beta_{21}(t)$ are all subject to fluctuation in time.

Considering that two species are constantly in the competition, and when a species suffers damage from another one by competition, another one could benefit, the duration time of density for species would also play an important role, we modified system (1.4) as the following

$$\begin{cases} \dot{x}_1(t) = x_1(t)\Big[K_1(t) - \alpha_1(t)x_1(t-\tau(t)) - \beta_{12}(t)x_2(t-\tau(t)) \\ \qquad\qquad -\gamma_1(t)x_1(t-\tau(t))x_2(t-\tau(t))\Big], \\ \dot{x}_2(t) = x_2(t)\Big[K_2(t) - \alpha_2(t)x_2(t-\tau(t)) - \beta_{21}(t)x_1(t-\tau(t)) \\ \qquad\qquad -\gamma_2(t)x_1(t-\tau(t))x_2(t-\tau(t))\Big], \end{cases} \tag{1.5}$$

where $\tau(t)$ is nonnegative constant which stands for the hunting delay. Many authors [44]–[47] have argued that discrete time models governed by difference equations are more appropriate to describe the dynamics relationship among populations than continuous ones when the populations have non-overlapping generations. Moreover, discrete time models can also provide efficient models of continuous ones for numerical simulations. In addition, we know that competition and cooperation systems of two enterprises based on ecosystem in the real world are continuously distributed by unpredictable forces which can result in changes in the system parameters such as growth rate, intrinsic growth rate and so on. Of practical interest in competition and cooperation systems of two enterprises is the question of whether or not a competition and cooperation system of two enterprises can withstand those unpredictable disturbances which persist for a finite period of time. In the language of control variables, we call the disturbance functions as control variables. Motivated by the analysis above, we can modify system (1.5) as follows

$$\begin{cases} x_1(n+1) = x_1(n)\exp\Big\{K_1(n) - \alpha_1(n)x_1(n-\tau(n)) \\ \quad -\beta_{12}(n)x_2(n-\tau(n)) - \gamma_1(n)x_1(n-\tau(n))x_2(n-\tau(n)) - \beta_1(n)u_1(n)\Big\}, \\ x_2(n+1) = x_2(n)\exp\Big\{K_2(n) - \alpha_2(n)x_2(n-\tau(n)) \\ \quad -\beta_{21}(n)x_1(n-\tau(n)) - \gamma_2(n)x_1(n-\tau(n))x_2(n-\tau(n)) - \beta_2(n)u_2(n)\Big\}, \\ \Delta u_1(n) = -\xi_1(n)u_1(n) + \eta_1(n)x_1(n), \\ \Delta u_2(n) = -\xi_2(n)u_2(n) + \eta_2(n)x_2(n), \end{cases} \tag{1.6}$$

where $x_1(n)$ and $x_2(n)$ denote the density of two competing species at the generation, respectively, and $u_i(n)(i = 1, 2)$ is the control variable. $K_i(n), \alpha_i(n), \gamma_i(n)$, $\beta_{12}(n), \beta_{21}(n)$ and $\tau(n)$ are bounded nonnegative sequences. To the authors' knowledge, it is first time to deal with system (1.6) with feedback control. We believe that this investigation on the permanence and global attractivity of enterprise clusters has important theoretical value and tremendous potential for application in administering process, economic performance and so on.

The main object of this paper is to investigate the permanence and global attractivity of model (1.6). In order to obtain our main results, throughout this paper, we assume that

$$\text{(H1) } 0 < K_i^l \le K_i^u, \quad 0 < \alpha_i^l \le \alpha_i^u \quad 0 < \beta_{12}^l \le \beta_{12}^u, \quad 0 < \beta_{21}^l \le \beta_{21}^u,$$
$$0 < \gamma_i^l \le \gamma_i^u, \quad 0 < \beta_i^l \le \beta_i^u (i = 1, 2).$$

Here, for any bounded sequence $\{f(n)\}$,

$$f^u = \sup_{n\in N}\{f(n)\} \quad \text{and} \quad f^l = \inf_{n\in N}\{f(n)\}.$$

Let $\tau^m = \sup_{n\in Z}\{\tau(n)\}, \tau^l = \inf_{n\in Z}\{\tau(n)\}$. We consider (1.6) together with the following initial conditions

$$x_i(\theta) = \varphi_i(\theta) \ge 0, \theta \in N[-\tau, 0] = \{-\tau, -\tau+1, \cdots, 0\}, \varphi_i(0) > 0. \tag{1.7}$$

It is not difficult to see that solutions of (1.6) and (1.7) are well defined for all $n \ge 0$ and satisfy

$$x_i(n) > 0, \text{for } n \in Z, i = 1, 2.$$

The remainder of the paper is organized as follows: in Section 2, basic definitions and lemmas are given, some sufficient conditions for the permanence of system (1.6) are established. In Section 3, a set of sufficient conditions which ensure the existence and stability of a unique globally attractive positive periodic solution of the system (when the time delays are equal to zero) are derived. In Section 4, two examples with their simulations are given to illustrate the feasibility and effectiveness of our results obtained is Section 2 and Section 3. Brief conclusions are drawn in Section 5.

2. Permanence

In order to obtain the main result of this paper, we shall first state the definition of permanence and several lemmas which will be useful in the proving the main result.

Definition 2.1. We say that system (1.6) is permanence if there are positive constants M and m such that for each positive solution $(x_1(n), x_2(n), u_1(n), u_2(n))$ of system (1.6) satisfies

$$m \leq \lim_{n\to+\infty} \inf x_i(n) \leq \lim_{n\to+\infty} \sup x_i(n) \leq M (i = 1, 2),$$

$$m \leq \lim_{n\to+\infty} \inf u_i(n) \leq \lim_{n\to+\infty} \sup u_i(n) \leq M (i = 1, 2).$$

Let us consider the following single species discrete model:

$$N(n + 1) = N(n) \exp(a(n) - b(n)N(n)), \tag{2.1}$$

where $\{a(n)\}$ and $\{b(n)\}$ are strictly positive sequences of real numbers defined for $n \in N = \{0, 1, 2, \cdots\}$ and $0 < a^l \leq a^u, 0 < b^l \leq b^u$. Similarly to the proofs of Propositions 1 and 3 in [54], we can obtain the following Lemma 2.1.

Lemma 2.1. *Any solution of system* (2.1) *with initial condition* $N(0) > 0$ *satisfies*

$$m \leq \lim_{n\to+\infty} \inf N(n) \leq \lim_{n\to+\infty} \sup N(n) \leq M,$$

where

$$M = \frac{1}{b^l} \exp(a^u - 1), m = \frac{a^l}{b^u} \exp(a^l - b^u M).$$

Let consider the first order difference equation

$$y(n + 1) = Ay(n) + B, n = 1, 2, \cdots , \tag{2.2}$$

where A and B are positive constants. Following Theorem 6.2 of Wang and Wang [55, page 125], we have the following Lemma 2.2.

Lemma 2.2 ([55])**.** *Assume that* $|A| < 1$, *for any initial value* $y(0)$, *there exists a unique solution* $y(n)$ *of* (2.2) *which can be expressed as follows:*

$$y(n) = A^n(y(0) - y^*) + y^*,$$

where $y^* = \frac{B}{1-A}$. *Thus, for any solution* $\{y(n)\}$ *of system* (2.2), $\lim_{n\to+\infty} y(n) = y^*$.

Lemma 2.3 ([55])**.** *Let* $n \in N_{n_0}^+ = \{n_0, n_0+1, \cdots, n_0+l, \cdots\}, r \geq 0$. *For any fixed* n, $g(n,r)$ *is a nondecreasing function with respect to* r, *and for* $n \geq n_0$, *the following inequalities hold:*

$$y(n+1) \leq g(n, y(n)), u(n+1) \geq g(n, u(n)).$$

If $y(n_0) \leq u(n_0)$, *then* $y(n) \leq u(n)$ *for all* $n \geq n_0$.

Proposition 2.1. *Assume that the condition* (H1) *holds, then*

$$\lim_{n\to+\infty} \sup x_i(n) \leq M_i, \lim_{n\to+\infty} \sup u_i(n) \leq U_i, i = 1, 2,$$

where

$$M_i = \frac{1}{\alpha_i^l} \exp\{K_i^u(\tau+1) - 1\}, U_i = \frac{\eta_i^u M_i}{\xi_i^l}(i = 1, 2).$$

Proof. Let $(x_1(n), x_2(n), u_1(n), u_2(n))$ be any positive solution of system (1.6) with the initial condition $(x_1(0), x_2(0), u_1(0), u_2(0))$. It follows from the first equation and the second equation of system (1.5) that

$$x_i(n+1) \leq x_i(n) \exp\{K_i(n)\}\,(i = 1, 2). \tag{2.3}$$

Let $x_i(n) = \exp\{y_i(n)\}(i = 1, 2)$, then (2.3) is equivalent to

$$y_i(n+1) - y_i(n) \leq K_i(n). \tag{2.4}$$

Summing both sides of (2.4) from $n - \tau(n)$ to $n - 1$, we have

$$\sum_{j=n-\tau(n)}^{n-1} (y_i(j+1) - y_i(j)) \leq \sum_{j=n-\tau(n)}^{n-1} K_i(j) \leq K_i^u \tau^m,$$

which leads to

$$y_i(n - \tau(n)) \geq y_i(n) - K_i^u \tau^m. \tag{2.5}$$

Then

$$x_i(n-\tau(n)) \geq x_i(n)\exp\{-K_i^u\tau^m\}. \tag{2.6}$$

Substituting (2.6) into the first and the second equations of system (1.5), it follows that

$$x_i(n+1) \leq x_i(n)\exp\Big\{K_i(n) - \alpha_i(n)\exp\{-K_i^u\tau^m\}x_i(n)\Big\}. \tag{2.7}$$

It follows from (2.7) and Lemma 2.1 that

$$\lim_{n\to+\infty}\sup x_i(n) \leq \frac{1}{\alpha_i^l}\exp\{K_i^u(\tau^m+1)-1\} := M_i. \tag{2.8}$$

For any positive constant $\varepsilon > 0$, it follows (2.8) that there exists a $N_1 > 0$ such that for all $n > N_1 + \tau$

$$x_i(n) \leq M_i + \varepsilon. \tag{2.9}$$

In view of the third and fourth equations of the system (1.5), we can obtain

$$\Delta u_i(n) \leq -\xi_i(n)u_i(n) + \eta_i(n)(M_i+\varepsilon)(i=1,2). \tag{2.10}$$

Then

$$u_i(n+1) \leq (1-\xi_i^l)u_i(n) + \eta_i^u(M_i+\varepsilon)(i=1,2). \tag{2.11}$$

Applying Lemma 2.2 and 2.3, it immediately follows that

$$\lim_{n\to+\infty}\sup u_i(n) \leq \frac{\eta_i^u(M_i+\varepsilon)}{\xi_i^l}(i=1,2). \tag{2.12}$$

Setting $\varepsilon \to 0$, it follows that

$$\lim_{n\to+\infty}\sup u_i(n) \leq \frac{\eta_i^u M_i}{\xi_i^l} := U_i(i=1,2). \tag{2.13}$$

This completes the proof of Proposition 2.1. $\square$

Theorem 2.1. *Let M_i and U_i are defined by* (2.8) *and* (2.13), *respectively. Assume that* (H1) *and*

$$(\mathrm{H2})\begin{cases} K_1^l > \beta_{12}^u M_2 + \gamma_1^u M_1 M_2 + \beta_1^u U_1, \\ K_2^l > \beta_{21}^u M_1 + \gamma_2^u M_1 M_2 + \beta_2^u U_2 \end{cases}$$

hold, then system (1.5) *is permanent.*

Proof. By applying Proposition 2.1, we are easily to see that to end the proof of Theorem 2.1, it is enough to show that under the conditions of Theorem 2.1,

$$\lim_{n\to+\infty}\inf x_2(n)\geq m_1,\ \lim_{n\to+\infty}\inf x_2(n)\geq m_2,$$

$$\lim_{n\to+\infty}\inf u_1(n)\geq v_1,\ \lim_{n\to+\infty}\inf u_2(n)\geq v_2.$$

In view of Proposition 2.1, for all $\varepsilon>0$, there exists a $N_2>0, N_2\in N$, for all $n>N_2$,

$$x_i(n)\leq M_i+\varepsilon, u_i(n)\leq U_i+\varepsilon, i=1,2. \tag{2.14}$$

It follows from system (1.5) and (2.14) that for all $n>N_2+\tau$,

$$\begin{cases} x_1(n+1)\geq x_1(n)\exp\Big\{K_1^l-\alpha_1^u(M_1+\varepsilon)-\beta_{12}^u(M_2+\varepsilon)\\ \quad -\gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon)-\beta_1^u(U_1+\varepsilon)\Big\},\\ x_2(n+1)\geq x_2(n)\exp\Big\{K_2^l-\alpha_2^u(M_2+\varepsilon)-\beta_{21}^u(M_1+\varepsilon)\\ \quad -\gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon)-\beta_2^u(U_2+\varepsilon)\Big\}. \end{cases} \tag{2.15}$$

Let $x_i(n)=\exp\{y_i(n)\}$, then (2.15) is equivalent to

$$\begin{cases} y_1(n+1)-y_1(n)\geq K_1^l-\alpha_1^u(M_1+\varepsilon)-\beta_{12}^u(M_2+\varepsilon)\\ \quad -\gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon)-\beta_1^u(U_1+\varepsilon),\\ y_2(n+1)-y_2(n)\geq K_2^l-\alpha_2^u(M_2+\varepsilon)-\beta_{21}^u(M_1+\varepsilon)\\ \quad -\gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon)-\beta_2^u(U_2+\varepsilon). \end{cases} \tag{2.16}$$

Summing both sides of both equations of (2.16) from $n-\tau(n)$ to $n-1$ leads to

$$\begin{cases} \sum\limits_{j=n-\tau(n)}^{n-1}(y_1(j+1)-y_1(j)) \ \geq \sum\limits_{j=n-\tau(n)}^{n-1}\Big[K_1^l-\alpha_1^u(M_1+\varepsilon)-\beta_{12}^u(M_2+\varepsilon)\\ \qquad -\gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon)-\beta_1^u(U_1+\varepsilon)\Big]\\ \quad \geq\Big[K_1^l-\alpha_1^u(M_1+\varepsilon)-\beta_{12}^u(M_2+\varepsilon)-\gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon)-\beta_1^u(U_1+\varepsilon)\Big]\tau^m,\\ \sum\limits_{j=n-\tau(n)}^{n-1}(y_2(j+1)-y_2(j)) \ \geq \sum\limits_{j=n-\tau(n)}^{n-1}\Big[K_2^l-\alpha_2^u(M_2+\varepsilon)-\beta_{21}^u(M_1+\varepsilon)\\ \qquad -\gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon)-\beta_2^u(U_2+\varepsilon)\Big]\\ \quad \geq\Big[K_2^l-\alpha_2^u(M_2+\varepsilon)-\beta_{21}^u(M_1+\varepsilon)-\gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon)-\beta_2^u(U_2+\varepsilon)\Big]\tau^m. \end{cases} \tag{2.17}$$

Then

$$\begin{cases} y_1(n-\tau(n)) \le y_1(n) - \Big[K_1^l - \alpha_1^u(M_1+\varepsilon) - \beta_{12}^u(M_2+\varepsilon) \\ \quad -\gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_1^u(U_1+\varepsilon)\Big]\tau^m, \\ y_2(n-\tau(n)) \le y_2(n) - \Big[K_2^l - \alpha_2^u(M_2+\varepsilon) - \beta_{21}^u(M_1+\varepsilon) \\ \quad -\gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_2^u(U_2+\varepsilon)\Big]\tau^m. \end{cases} \tag{2.18}$$

Thus

$$\begin{cases} x_1(n-\tau(n)) \le x_1(n)\exp\Big\{-\Big[K_1^l - \alpha_1^u(M_1+\varepsilon) - \beta_{12}^u(M_2+\varepsilon) \\ \quad -\gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_1^u(U_1+\varepsilon)\Big]\tau^m\Big\}, \\ x_2(n-\tau(n)) \le x_2(n)\exp\Big\{-\Big[K_2^l - \alpha_2^u(M_2+\varepsilon) - \beta_{21}^u(M_1+\varepsilon) \\ \quad -\gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_2^u(U_2+\varepsilon)\Big]\tau^m\Big\}. \end{cases} \tag{2.19}$$

Substituting (2.19) into the first and second equation of (1.5), we have

$$\begin{cases} x_1(n+1) \ge x_1(n)\exp\Big\{K_1^l - \beta_{12}^u(M_2+\varepsilon) - \gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon) \\ \quad -\beta_1^u(U_1+\varepsilon) - \alpha_1^u x_1(n)\exp\Big\{-\Big[K_1^l - \alpha_1^u(M_1+\varepsilon) - \beta_{12}^u(M_2+\varepsilon) \\ \quad\quad -\gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_1^u(U_1+\varepsilon)\Big]\tau^m\Big\}\Big\}, \\ x_2(n+1) \ge x_2(n)\exp\Big\{K_2^l - \beta_{21}^u(M_1+\varepsilon) - \gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon) \\ \quad -\beta_2^u(U_2+\varepsilon) - \alpha_2^u x_2(n)\exp\Big\{-\Big[K_2^l - \alpha_2^u(M_2+\varepsilon) - \beta_{21}^u(M_1+\varepsilon) \\ \quad\quad -\gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_2^u(U_2+\varepsilon)\Big]\tau^m\Big\}\Big\}. \end{cases} \tag{2.20}$$

By applying Lemma 2.1 and 2.3, it immediately follows that

$$\lim_{n\to+\infty}\inf x_1(n) \ge m_1^\varepsilon, \ \lim_{n\to+\infty}\inf x_2(n) \ge m_2^\varepsilon, \tag{2.21}$$

where

$$\begin{cases} m_1^\varepsilon = \dfrac{K_1^l - \beta_{12}^u(M_2+\varepsilon) - \gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_1^u(U_1+\varepsilon)}{\alpha_1^u\exp\Big\{-\Big[K_1^l - \alpha_1^u(M_1+\varepsilon) - \beta_{12}^u(M_2+\varepsilon) - \gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_1^u(U_1+\varepsilon)\Big]\tau^m\Big\}} \\ \quad \times\exp\Big\{K_1^l - \beta_{12}^u(M_2+\varepsilon) - \gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_1^u(U_1+\varepsilon) - \alpha_1^u\exp\Big\{ \\ \quad\quad -\Big[K_1^l - \alpha_1^u(M_1+\varepsilon) - \beta_{12}^u(M_2+\varepsilon) - \gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_1^u(U_1+\varepsilon)\Big]\tau^m\Big\}M_1\Big\}, \\ m_2^\varepsilon = \dfrac{K_2^l - \beta_{21}^u(M_1+\varepsilon) - \gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_2^u(U_2+\varepsilon)}{\alpha_2^u\exp\Big\{-\Big[K_2^l - \alpha_2^u(M_2+\varepsilon) - \beta_{21}^u(M_1+\varepsilon) - \gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_2^u(U_2+\varepsilon)\Big]\tau^m\Big\}} \\ \quad \times\exp\Big\{K_2^l - \beta_{21}^u(M_1+\varepsilon) - \gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_2^u(U_2+\varepsilon) - \alpha_2^u\exp\Big\{ \\ \quad\quad -\Big[K_2^l - \alpha_2^u(M_2+\varepsilon) - \beta_{21}^u(M_1+\varepsilon) - \gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon) - \beta_2^u(U_2+\varepsilon)\Big]\tau^m\Big\}M_2\Big\}. \end{cases} \tag{2.22}$$

Setting $\varepsilon \to 0$ in (2.22), then

$$\lim_{n\to+\infty} \inf x_1(n) \geq m_1, \lim_{n\to+\infty} \inf x_2(n) \geq m_2, \tag{2.23}$$

where

$$\begin{cases} m_1^\varepsilon = \dfrac{K_1^l - \beta_{12}^u M_2 - \gamma_1^u M_1 M_2 - \beta_1^u U_1}{\alpha_1^u \exp\Big\{ - \Big[K_1^l - \alpha_1^u M_1 - \beta_{12}^u M_2 - \gamma_1^u M_1 M_2 - \beta_1^u U_1\Big]\tau^m\Big\}} \\ \quad \times \exp\Bigg\{K_1^l - \beta_{12}^u M_2 - \gamma_1^u M_1 M_2 - \beta_1^u U_1 - \alpha_1^u \exp\Big\{ \\ \quad -\Big[K_1^l - \alpha_1^u M_1 - \beta_{12}^u M_2 - \gamma_1^u M_1 M_2 - \beta_1^u U_1\Big]\tau^m\Big\} M_1\Bigg\}, \\ m_2^\varepsilon = \dfrac{K_2^l - \beta_{21}^u M_1 - \gamma_2^u M_1 M_2 - \beta_2^u U_2}{\alpha_2^u \exp\Big\{ - \Big[K_2^l - \alpha_2^u M_2 - \beta_{21}^u M_1 - \gamma_2^u M_1 M_2 - \beta_2^u U_2\Big]\tau^m\Big\}} \\ \quad \times \exp\Bigg\{K_2^l - \beta_{21}^u M_1 - \gamma_2^u M_1 M_2 - \beta_2^u U_2 - \alpha_2^u \exp\Big\{ \\ \quad -\Big[K_2^l - \alpha_2^u M_2 - \beta_{21}^u M_1 - \gamma_2^u M_1 M_2 - \beta_2^u U_2\Big]\tau^m\Big\} M_2\Bigg\}. \end{cases} \tag{2.24}$$

Without loss of generality, we assume that $\varepsilon < \frac{1}{2}\min\{m_1, m_2\}$. For any positive constant ε small enough, it follows from (2.23) that there exists enough large $N_3 > N_2 + \tau$ such that

$$x_1(n) \geq m_1 - \varepsilon, x_2(n) \geq m_2 - \varepsilon \tag{2.25}$$

for any $n \geq N_3$. From the third and fourth equations of system (1.5) and (2.25), we can derive that

$$\Delta u_i(n) \geq -\xi_i(n)u_i(n) + \eta_i(n)(m_i - \varepsilon), i = 1, 2. \tag{2.26}$$

Hence

$$u_i(n+1) \geq (1 - \xi_i^u)u_i(n) + \eta_i^l(m_i - \varepsilon), i = 1, 2. \tag{2.27}$$

By applying Lemma 2.1 and 2.2, it immediately follows that

$$\lim_{n\to+\infty} \inf u_i(n) \geq \frac{\eta_i^l(m_i - \varepsilon)}{\xi_i^u}, i = 1, 2. \tag{2.28}$$

Setting $\varepsilon \to 0$ in the above inequality leads to

$$\lim_{n\to+\infty} \inf u_i(n) \geq \frac{\eta_i^l m_i}{\xi_i^u} := U_i^l, i = 1, 2. \tag{2.29}$$

This completes the proof of Theorem 2.1. $\square$

3. Existence and stability of periodic solution

In this section, we will consider the stability of system (1.5) under the assumption $\tau(n) = 0$, namely, we consider the following system

$$\begin{cases} x_1(n+1) & = x_1(n) \exp\Big\{K_1(n) - \alpha_1(n)x_1(n) - \beta_{12}(n)x_2(n) \\ & \qquad -\gamma_1(n)x_1(n)x_2(n) - \beta_1(n)u_1(n)\Big\}, \\ x_2(n+1) & = x_2(n) \exp\Big\{K_2(n) - \alpha_2(n)x_2(n) - \beta_{21}(n)x_1(n) \\ & \qquad -\gamma_2(n)x_1(n)x_2(n) - \beta_2(n)u_2(n)\Big\}, \\ \Delta u_1(n) & = -\xi_1(n)u_1(n) + \eta_1(n)x_1(n), \\ \Delta u_2(n) & = -\xi_2(n)u_2(n) + \eta_2(n)x_2(n), \end{cases} \tag{3.1}$$

Throughout this section, we always assume that $K_i(n), \gamma_i(n), \xi_i(n), \eta_i(n) (i = 1, 2)$, $\beta_{12}(n), \beta_{21}(n)$ are all bounded negative periodic sequences with a common periodic ω and satisfy

$$0 < \xi_i(n) < 1, n \in N \cap [0, \omega], i = 1, 2. \tag{3.2}$$

Also it is assumed that the initial conditions of (3.1) are of the form

$$x_i(0) > 0, u_i(0) > 0, i = 1, 2. \tag{3.3}$$

Applying the similar way, under some conditions, we can obtain the permanence of system (3.1). We still let M_i and U_i be the upper bound of $\{x_i(n)\}$ and $\{u_i(n)\}$, and m_i and U_i^l be the lower bound of $\{x_i(n)\}$ and $\{u_i(n)\}$.

Theorem 3.1. *In addition to* (3.2), *assume that* (H1) *and*

$$\text{(H2)} \begin{cases} K_1^l > \beta_{12}^u M_2 + \gamma_1^u M_1 M_2 + \beta_1^u U_1, \\ K_2^l > \beta_{21}^u M_1 + \gamma_2^u M_1 M_2 + \beta_2^u U_2 \end{cases}$$

hold, then system (3.1) *has a periodic solution denoted by* $\{\bar{x}_1(n), \bar{x}_2(n), \bar{u}_1(n), \bar{u}_2(n)\}$.

Proof. Let $\Omega = \{(x_1, x_2, u_1, u_2) | m_i \le x_i \le M_i, U_i^l \le u_i \le U_i, i = 1, 2\}$. It is easy to see that Ω is an invariant set of system (3.1). Then we can define a mapping F on Ω by

$$F(x_1(0), x_2(0), u_1(0), u_2(0)) = (x_1(\omega), x_2(\omega), u_1(\omega), u_2(\omega)) \tag{3.4}$$

for $(x_1(0), x_2(0), u_1(0), u_2(0)) \in \Omega$. Obviously, F depends continuously on $(x_1(0), x_2(0), u_1(0), u_2(0))$. Thus F is continuous and maps a compact set Ω into itself. Therefore, F has a fixed point $(\bar{x}_1(n), \bar{x}_2(n), \bar{u}_1(n), \bar{u}_2(n))$. So we can conclude that the solution $(\bar{x}_1(n), \bar{x}_2(n), \bar{u}_1(n), \bar{u}_2(n))$ passing through $(\bar{x}_1, \bar{x}_2, \bar{u}_1, \bar{u}_2)$ is a periodic solution of system (3.1). The proof of Theorem 3.1 is complete. □

Next, we investigate the global stability property of the periodic solution obtained in Theorem 3.1.

Theorem 3.2. *In addition to the conditions of Theorem 3.1, assume that the following condition* (H3) *holds,*

$$\text{(H3)}\begin{cases} \ell_1 = \max\Big\{|1-\alpha_1^l m_1 - \gamma_1^l m_1 m_2|, |1-\alpha_1^u M_1 - \gamma_1^u M_1 M_2|\Big\} \\ \qquad +\beta_{12}^u M_2 + \gamma_1^u M_1 M_2 + \beta_1^u < 1, \\ \ell_2 = \max\Big\{|1-\alpha_2^l m_2 - \gamma_2^l m_1 m_2|, |1-\alpha_2^u M_2 - \gamma_2^u M_1 M_2|\Big\} \\ \qquad +\beta_{21}^u M_1 + \gamma_2^u M_1 M_2 + \beta_2^u < 1, \\ \ell_3 = (1-\xi_1^l) + \eta_1^u M_1 < 1, \\ \ell_4 = (1-\xi_2^l) + \eta_2^u M_2 < 1, \end{cases}$$

then the ω periodic solution $(\bar{x}_1(n), \bar{x}_2(n), \bar{u}_1(n), \bar{u}_2(n))$ obtained in Theorem 3.1 is globally attractive.

Proof. Assume that $(x_1(n), x_2(n), u_1(n), u_2(n))$ is any positive solution of system (3.1). Let

$$x_i(n) = \bar{x}_i(n)\exp\{y_i(n)\}, u_i(n) = \bar{u}_i(n) + v_i(n), i = 1, 2. \tag{3.5}$$

To complete the proof, it suffices to show

$$\lim_{n\to\infty} y_i(n) = 0, \lim_{n\to\infty} v_i(n) = 0, i = 1, 2. \tag{3.6}$$

Since

$$\begin{aligned} y_1(n+1) = {} & y_1(n) - \alpha_1(n)\bar{x}_1(n)[\exp(y_1(n)) - 1] \\ & - \beta_{12}(n)\bar{x}_2(n)[\exp(y_2(n)) - 1] \\ & - \gamma_1(n)\bar{x}_1(n)\bar{x}_2(n)[\exp(y_1(n) + y_2(n)) - 1] - \beta_1(n)v_1(n) \\ = {} & y_1(n) - \alpha_1(n)\bar{x}_1(n)\exp\{\theta_1(n)y_1(n)\}y_1(n) \\ & - \beta_{12}(n)\bar{x}_2(n)\exp\{\theta_2(n)y_2(n)\}y_2(n) \\ & - \gamma_1(n)\bar{x}_1(n)\bar{x}_2(n)\exp\{\theta_3(n)(y_1(n) + y_2(n))\} \\ & \times (y_1(n) + y_2(n)) - \beta_1(n)v_1(n), \end{aligned} \tag{3.7}$$

where $\theta_i(n) \in (0,1), i = 1,2,3$. In a similar way, we get

$$\begin{aligned} y_2(n+1) = y_2(n) &- \alpha_2(n)\bar{x}_2(n)\exp\{\theta_4(n)y_2(n)\}y_2(n) \\ &- \beta_{21}(n)\bar{x}_1(n)\exp\{\theta_5(n)y_1(n)\}y_1(n) \\ &- \gamma_2(n)\bar{x}_1(n)\bar{x}_2(n)\exp\{\theta_6(n)(y_1(n)+y_2(n))\} \\ &\times (y_1(n)+y_2(n)) - \beta_2(n)v_2(n), \end{aligned} \tag{3.8}$$

where $\theta_j(n) \in (0,1), j = 4,5,6$.

Also, one has

$$\begin{aligned} v_1(n+1) &= (1-\xi_1(n))v_1(n) + \eta_1(n)\bar{x}_1(n)[\exp\{y_1(n)\}-1] \\ &= (1-\gamma_1(n))v_1(n) + \eta_1(n)\bar{x}_1(n)\exp\{\theta_7(n)y_1(n)\}y_1(n), \end{aligned} \tag{3.9}$$

$$\begin{aligned} v_2(n+1) &= (1-\xi_2(n))v_2(n) + \eta_2(n)\bar{x}_2(n)[\exp\{y_2(n)\}-1] \\ &= (1-\gamma_2(n))v_2(n) + \eta_2(n)\bar{x}_2(n)\exp\{\theta_8(n)y_2(n)\}y_2(n). \end{aligned} \tag{3.10}$$

By (H3), we can choose a $\varepsilon > 0$ such that

$$\begin{cases} \ell_1^\varepsilon = \max\Big\{|1-\alpha_1^l(m_1-\varepsilon) - \gamma_1^l(m_1-\varepsilon)(m_2-\varepsilon)|, \\ \qquad |1-\alpha_1^u(M_1+\varepsilon) - \gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon)|\Big\} \\ \qquad +\beta_{12}^u(M_2+\varepsilon) + \gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon) + \beta_1^u < 1 \\ \ell_2^\varepsilon = \max\Big\{|1-\alpha_2^l(m_2-\varepsilon) - \gamma_2^l(m_1-\varepsilon)(m_2-\varepsilon)|, \\ \qquad |1-\alpha_2^u(M_2+\varepsilon) - \gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon)|\Big\} \\ \qquad +\beta_{21}^u(M_1+\varepsilon) + \gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon) + \beta_2^u < 1, \\ \ell_3^\varepsilon = (1-\xi_1^l) + \eta_1^u(M_1+\varepsilon) < 1, \\ \ell_4^\varepsilon = (1-\xi_2^l) + \eta_2^u(M_2+\varepsilon) < 1, \end{cases} \tag{3.11}$$

In view of Proposition 2.1 and Theorem 2.1, there exists $N_4 > N_3$ such that

$$m_i - \varepsilon \le x_i(n), \bar{x}_i(n) \le M_i + \varepsilon, \text{for } n \ge N_5, i = 1,2. \tag{3.12}$$

It follows from (3.7) and (3.8) that

$$\begin{aligned} y_1(n+1) \le \max\big\{&|1-\alpha_1^l(m_1-\varepsilon) - \gamma_1^l(m_1-\varepsilon)(m_2-\varepsilon)|, \\ &|1-\alpha_1^u(M_1+\varepsilon) - \gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon)|\big\}|y_1(n)| \\ &+ [\beta_{12}^u(M_2+\varepsilon) + \gamma_1^u(M_1+\varepsilon)(M_2+\varepsilon)]|y_2(n)| + \beta_1^u|v_1(n)|, \end{aligned} \tag{3.13}$$

$$\begin{aligned}y_2(n+1) \le \max\big\{&|1-\alpha_2^l(m_2-\varepsilon)-\gamma_2^l(m_1-\varepsilon)(m_2-\varepsilon)|,\\ &|1-\alpha_2^u(M_2+\varepsilon)-\gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon)|\big\}|y_2(n)|\\ &+[\beta_{21}^u(M_1+\varepsilon)+\gamma_2^u(M_1+\varepsilon)(M_2+\varepsilon)]|y_1(n)|+\beta_2^u|v_2(n)|, \quad (3.14)\end{aligned}$$

Also, for $n > N_4$, one has

$$v_1(n+1) \le (1-\gamma_1^l)|v_1(n)| + \eta_1^u(M_1+\varepsilon)|y_1(n)|, \quad (3.15)$$

$$v_2(n+1) \le (1-\gamma_2^l)|v_2(n)| + \eta_2^u(M_2+\varepsilon)|y_2(n)|. \quad (3.16)$$

Let $\ell = \max\{\ell_1^\varepsilon, \ell_2^\varepsilon, \ell_3^\varepsilon, \ell_4^\varepsilon\}$, then $0 < \ell < 1$. It follows from (3.13)–(3.16) that

$$\begin{aligned}&\max\{|y_1(n+1)|, |y_2(n+1)|, |v_1(n+1)|, |v_2(n+1)|\}\\ &\quad \le \chi \max\{|y_1(n)|, |y_2(n)|, |v_1(n)|, |v_2(n)|\} \quad (3.17)\end{aligned}$$

for $n > N_4$. Then we get

$$\begin{aligned}&\max\{|y_1(n)|, |y_2(n)|, |v_1(n)|, |v_2(n)|\}\\ &\quad \le \chi^{n-N_4} \max\{|y_1(N_4)|, |y_2(N_4)|, |v_1(N_4)|, |v_2(N_4)|\}. \quad (3.18)\end{aligned}$$

Thus

$$\lim_{n\to\infty} y_i(n) = 0, \lim_{n\to\infty} v_i(n) = 0, i = 1, 2. \quad (3.19)$$

This completes the proof. □

4. Examples

In this section, we give two examples with their numerical simulations to illustrate the feasibility of our results.

Example 4.1. Consider the following system

$$\begin{cases} x_1(n+1) &= x_1(n)\exp\Big\{K_1(n)-\alpha_1(n)x_1(n-\tau(n))-\beta_{12}(n)x_2(n-\tau(n))\\ &\quad -\gamma_1(n)x_1(n-\tau(n))x_2(n-\tau(n))-\beta_1(n)u_1(n)\Big\},\\ x_2(n+1) &= x_2(n)\exp\Big\{K_2(n)-\alpha_2(n)x_2(n-\tau(n))-\beta_{21}(n)x_1(n-\tau(n))\\ &\quad -\gamma_2(n)x_1(n-\tau(n))x_2(n-\tau(n))-\beta_2(n)u_2(n)\Big\},\\ \Delta u_1(n) &= -\gamma_1(n)u_1(n)+\eta_1(n)x_1(n),\\ \Delta u_2(n) &= -\gamma_2(n)u_2(n)+\eta_2(n)x_2(n), \end{cases} \quad (4.1)$$

where $K_1(n) = 2 + \cos(n), K_2(n) = 2 + \sin(n), \alpha_1(n) = 16 + \sin(n), \alpha_2(n) = 16 + \sin(n), \xi_1(n) = 1 - 0.8\cos(n), \xi_2(n) = 1 - 0.7\sin(n), \eta_1(n) = 0.005 + 0.005\cos(n)$, $\eta_2(n) = 0.005 + 0.005\sin(n), \beta_{12}(n) = 0.3 + 0.2\sin(n), \beta_{21}(n) = 0.2 + 0.1\cos(n)$, $\gamma_1(n) = 0.2 + 0.2\sin(n), \gamma_2(n) = 0.1 + 0.2\cos(n), \beta_1(n) = 0.1 + 0.1\sin(n), \beta_2(n) = 0.1 + 0.1\cos(n), \tau(n) = 0.1$. Then $K_1^l = 1, K_2^l = 1, K_1^u = 3, K_2^u = 3, \beta_{12}^u = 0.5, \beta_{21}^u = 0.3, \beta_1^u = 0.2, \beta_2^u = 0.2, \gamma_1^u = 0.4, \gamma_2^u = 0.3, \xi_1^l = 0.2, \xi_2^l = 0.3, \alpha_1^l = 15, \alpha_2^l = 15$. Thus $M_1 \approx 0.6234, M_2 \approx 0.6234, m_1 \approx 0.3032, m_2 \approx 0.3241, U_1 \approx 0.0312, U_2 \approx 0.0208$, $\beta_{12}^u M_2 + \gamma_1^u M_1 M_2 + \beta_1^u U_1 \approx 0.47, \beta_{21}^u M_1 + \gamma_2^u M_1 M_2 + \beta_2^u U_2 \approx 0.31$. One can check that all the conditions in Theorem 2.1 are satisfied. Then we can conclude that system (4.1) is permanent which is shown in Figures 1–4.

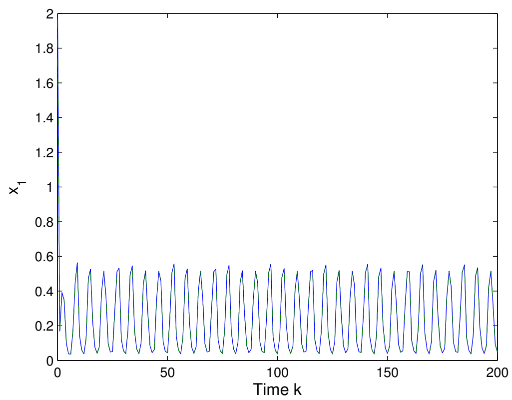

Figure 1. Dynamical behavior of system (4.1): times series of x_1.

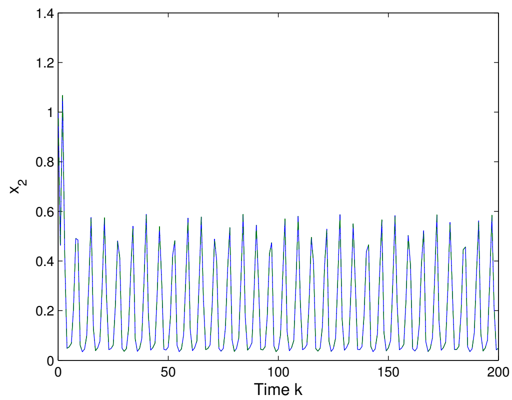

Figure 2. Dynamical behavior of system (4.1): times series of x_2.

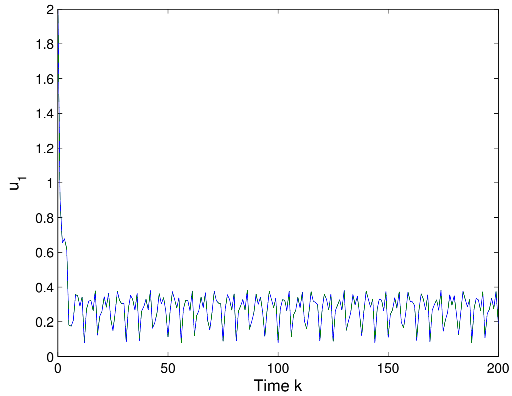

Figure 3. Dynamical behavior of system (4.1): times series of u_1.

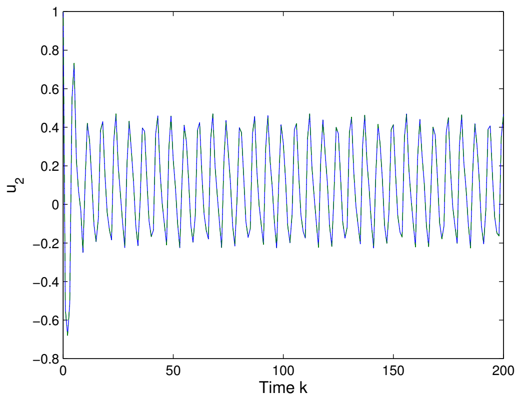

Figure 4. Dynamical behavior of system (4.1): times series of u_2.

Example 4.2. Consider the following system

$$\begin{cases} x_1(n+1) & = x_1(n)\exp\Big\{K_1(n) - \alpha_1(n)x_1(n) - \beta_{12}(n)x_2(n) \\ & \quad -\gamma_1(n)x_1(n)x_2(n) - \beta_1(n)u_1(n)\Big\}, \\ x_2(n+1) & = x_2(n)\exp\Big\{K_2(n) - \alpha_2(n)x_2(n) - \beta_{21}(n)x_1(n) \\ & \quad -\gamma_2(n)x_1(n)x_2(n) - \beta_2(n)u_2(n)\Big\}, \\ \Delta u_1(n) & = -\gamma_1(n)u_1(n) + \eta_1(n)x_1(n), \\ \Delta u_2(n) & = -\gamma_2(n)u_2(n) + \eta_2(n)x_2(n), \end{cases} \tag{4.2}$$

where $K_1(n) = 2 + \sin(n), K_2(n) = 2 + \cos(n), \alpha_1(n) = 11 + \sin(n), \alpha_2(n) = 11+\cos(n), \xi_1(n) = 1-0.6\cos(n), \xi_2(n) = 1-0.5\sin(n), \eta_1(n) = 0.05+0.05\cos(n), \eta_2(n) = 0.05 + 0.05\sin(n), \beta_{12}(n) = 0.1 + 0.2\cos(n), \beta_{21}(n) = 0.2 + 0.1\sin(n), \gamma_1(n) = 0.1+0.1\sin(n), \gamma_2(n) = 0.1+0.2\cos(n), \beta_1(n) = 0.1+0.2\sin(n), \beta_2(n) = 0.2+0.1\cos(n)$. Then $K_1^l = 1, K_2^l = 1, K_1^u = 3, K_2^u = 3, \beta_{12}^u = 0.3, \beta_{21}^u = 0.3, \beta_1^u = 0.3, \beta_2^u = 0.3, \gamma_1^u = 0.2, \gamma_2^u = 0.3, \xi_1^l = 0.4, \xi_2^l = 0.5, \alpha_1^l = 10, \alpha_2^l = 10$. Thus $M_1 \approx 0.7389, M_2 \approx 0.7389, m_1 \approx 0.2171, m_2 \approx 0.2004, U_1 \approx 0.1847, U_2 \approx 0.1478, \beta_{12}^u M_2 + \gamma_1^u M_1 M_2 + \beta_1^u U_1 \approx 0.3863, \beta_{21}^u M_1 + \gamma_2^u M_1 M_2 + \beta_2^u U_2 \approx 0.4298, \ell_1 \approx 0.4306, \ell_2 \approx 0.3277, \ell_3 \approx 0.6739, \ell_4 \approx 0.4298$. One can check that all the conditions in Theorem 3.1 are fulfilled. Then we can conclude that the periodic solution of system (4.2) is globally attractive which is illustrated in Figures 5–8.

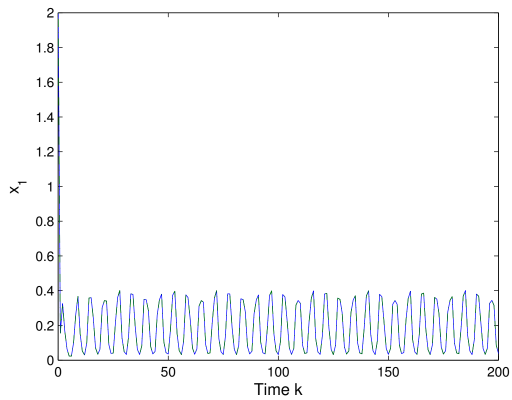

Figure 5. Dynamical behavior of system (4.2): times series of x_1.

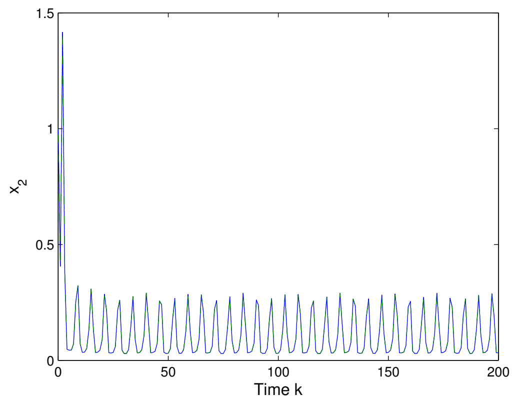

Figure 6. Dynamical behavior of system (4.2): times series of x_2.

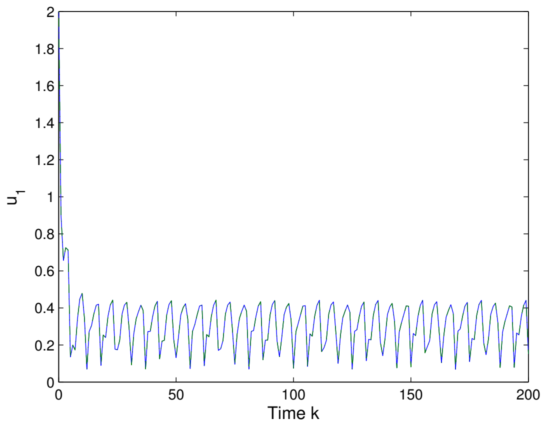

Figure 7. Dynamical behavior of system (4.2): times series of u_1.

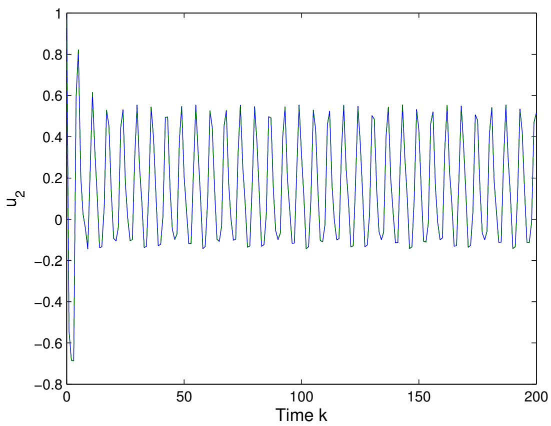

Figure 8. Dynamical behavior of system (4.2): times series of u_2.

5. Conclusions

In the present paper, we proposed a discrete two-species competitive model of plankton allelopathy with delays and feedback controls. Applying the difference inequality theory, we obtain some sufficient conditions which guarantee that the permanence of the system is established. It is shown that under some suitable conditions, the competition of two species can keep a dynamical balance. Thus we can conclude that feedback control effect and time delays are important factors to decide the co-existence of two species. Moreover, we also derive a set of sufficient conditions which ensure the existence and stability of unique globally attractive periodic solution of the system without time delays. Our results are new and complement the existing results in [29]–[30], [43].

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