

Title: On the variety of bands in completely regular semigroups

Author(s): Mario Petrich

Completely regular semigroups, enriched by the unary operation of inversion within their maximal subgroups, form a variety $\mathcal{CR}$ whose lattice of subvarieties is denoted by $\mathcal{L}(\mathcal{CR})$. Its subvariety $\mathcal{B}$ of all bands plays a seminal role in any study of the structure of $\mathcal{L}(\mathcal{CR})$. We present some new aspects of $\mathcal{B}$ relative to both $\mathcal{CR}$ and the variety $\mathcal{CS}$ of completely simple semigroups.

Since $\mathcal{B}$ is neutral in $\mathcal{L}(\mathcal{CR})$, the latter is a subdirect product of the lattice $(\mathcal{B}]$ of subvarieties of $\mathcal{B}$ and the lattice $[\mathcal{B})$ of supervarieties of $\mathcal{B}$. We determine the precise image of $\mathcal{L}(\mathcal{CR})$ in $(\mathcal{B}] \times [\mathcal{B})$.

For the relation $\mathbf{B}^\vee$ defined on $\mathcal{L}(\mathcal{CR})$ by $\mathcal{U}\mathbf{B}^\vee\mathcal{V}$ if $\mathcal{U} \vee \mathcal{B} = \mathcal{V} \vee \mathcal{B}$, we prove that each $\mathbf{B}^\vee$ -class is embeddable into $(\mathcal{B}]$.

We establish several results concerning the variety $\mathcal{CS}$ in the context of the relation $\mathbf{B}^\vee$ and the structure of the lattice $\mathcal{L}(\mathcal{CS})$.

Address:

Mario Petrich
21420 Bol
Brač
Croatia