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Title: The groups $K_1(\mathbb{S}_n, \mathfrak{p})$ of the algebra of one-sided inverses of a polynomial algebra

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The algebra $\mathbb{S}_n$ of one-sided inverses of a polynomial algebra P_n in n variables is obtained from P_n by adding commuting, *left* (but not two-sided) inverses of the canonical generators of the algebra P_n . The algebra $\mathbb{S}_n$ is a noncommutative, non-Noetherian algebra of classical Krull dimension $2n$ and of global dimension n , and is not a domain. If the ground field K has characteristic zero, then the algebra $\mathbb{S}_n$ is canonically isomorphic to the algebra $K\langle \frac{\partial}{\partial x_1}, \dots, \frac{\partial}{\partial x_n}, f_1, \dots, f_n \rangle$ of scalar integro-differential operators. It is proved that $K_1(\mathbb{S}_n) \simeq K^*$. The main idea is to show that the group $\mathrm{GL}_\infty(\mathbb{S}_n)$ is generated by K^* , the group of elementary matrices $E_\infty(\mathbb{S}_n)$ and $(n-2)2^{n-1} + 1$ explicit (tricky) matrices, and then to prove that all the matrices are elementary. For each nonzero idempotent prime ideal $\mathfrak{p}$ of height m of the algebra $\mathbb{S}_n$, it is proved that

$$K_1(\mathbb{S}_n, \mathfrak{p}) \simeq \begin{cases} K^*, & \text{if } m = 1, \\ \mathbb{Z}^{\frac{m(m-1)}{2}} \times K^{*m} & \text{if } m > 1. \end{cases}$$

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