

Title: Generalized symmetric $*$ -rings and Jacobson's Lemma for Moore–Penrose inverse

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It is well known as Jacobson's Lemma that $1 - ba$ is invertible in a ring if so is $1 - ab$. Moreover, if $c = (1 - ab)^{-1}$, then $(1 - ba)^{-1} = 1 + bca$. However, the analogous statement for Moore–Penrose inverse in a $*$ -ring is not true in general. Note that Jacobson's Lemma for Moore–Penrose inverse holds true in a symmetric $*$ -ring. In this paper, we study symmetric $*$ -rings and introduce the notion of a generalized symmetric $*$ -ring. A $*$ -ring R is called generalized symmetric if $1 - (u^* - u)^2$ is invertible for all units u in R . When $1 - ab$ is Moore–Penrose invertible in such a ring, we provide sufficient and necessary conditions under which $1 - ba$ has a Moore–Penrose inverse $(1 - ba)^\dagger$ and give a formula for $(1 - ba)^\dagger$.

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