

Title: On characterizations and topology of regular semimetric spaces

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We examine semimetric spaces, i.e., spaces fulfilling two of the axioms of a metric space excluding the triangle inequality. We give an overview of many substitutes of the triangle condition which have appeared in the mathematical literature until now and examine the relations between them. We review and compare various ways of introducing a topology in a semimetric space and investigate its metrizability. We also answer some open questions posed by Bessenyei and Páles, Dung and Hang, and Khamsi and Hussain. In particular, we show that a semimetric space is regular in the sense of Bessenyei and Páles if and only if it is uniformly metrizable. This substantially improves a very recent result of Dung and Hang. We also describe a general method of constructing semimetries satisfying the so-called c -relaxed polygonal inequality and having the property that every open ball is not open and, simultaneously, every closed ball is not closed.

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