

Title: Mean invariance identity

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For a continuous and increasing function f in a real interval I , and a bivariable mean P defined in I^2 , we prescribe a pair of bivariable means M and N such that the quasiarithmetic mean A_f generated by f is invariant with respect to the mean-type mapping (M, N) . This allows to find effectively the limit of the iterates of the mean-type mapping (M, N) . The means M and N are equal iff P is the arithmetic mean A ; they are symmetric iff so is P . Treating f and P as the parameters, we obtain the family of all pairs of means (M, N) such that the quasiarithmetic mean A_f is invariant with respect to (M, N) . In particular, we indicate the function f and the mean P such that the invariance identity $A_f \circ (M, N) = A_f$ coincides with the equality $G \circ (H, A)$, where G and H are the geometric and harmonic means, equivalent to the classical Pythagorean harmony proportion.

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