

Gradient estimates for some evolution equations on complete smooth metric measure spaces

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Abstract. In this paper, we consider the following general evolution equation

$$u_t = \Delta_f u + au \log^\alpha u + bu$$

on a smooth metric measure space $(M^n, g, e^{-f} dv)$. We give a local gradient estimate of Souplet–Zhang type for positive smooth solutions of this equation provided that the Bakry–Émery curvature is bounded from below. When f is constant, we investigate the general evolution equation on compact Riemannian manifolds with nonconvex boundary satisfying an *interior rolling R -ball* condition. We show a gradient estimate of Hamilton type on such manifolds.

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