

Sincov's inequalities on topological spaces

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Abstract. Assume that X is a non-empty set, and T and S are real or complex mappings defined on the product $X \times X$. Additive and multiplicative Sincov's equations are:

$$T(x, z) = T(x, y) + T(y, z), \quad x, y, z \in X$$

and

$$S(x, z) = S(x, y) \cdot S(y, z), \quad x, y, z \in X,$$

respectively. In the present paper, we study three related inequalities. We begin with functional inequality

$$G(x, z) \leq G(x, y) \cdot G(y, z), \quad x, y, z \in X,$$

and assume that X is a topological space and $G: X \times X \rightarrow \mathbb{R}$ is a continuous mapping. In some of our statements a considerably weaker regularity than continuity of G is needed. Next, we study the reverse inequality:

$$F(x, z) \geq F(x, y) \cdot F(y, z), \quad x, y, z \in X,$$

as well as the additive inequality:

$$H(x, z) \leq H(x, y) + H(y, z), \quad x, y, z \in X.$$

A corollary for generalized metric is derived.

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