

## Reducibility of polynomials after a polynomial substitution

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**Abstract.** We prove that for any field  $K$  and any polynomial  $f \in K[x]$  of degree  $d$  which is irreducible over  $K$ , there exists a polynomial  $g \in K[x]$  of degree  $d-1$  such that the composition polynomial  $f(g(x))$  is reducible over  $K$ . This answers a corresponding question recently raised by Ulas. We also characterize all quartic polynomials  $f \in K[x]$ , where  $K$  is a field of characteristic zero, for which  $f(g(x))$  remains irreducible over  $K$  under any quadratic substitution  $g \in K[x]$ . This characterization is given in terms of  $K$ -rational points on an elliptic curve of genus 1. As a corollary, we show that the polynomial  $g(x)^4 + 1$  is irreducible over  $\mathbb{Q}$  for any quadratic polynomial  $g \in \mathbb{Q}[x]$ .

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