

Congruences involving binomial coefficients and Apéry-like numbers

By ZHI-HONG SUN (Huaian)

Abstract. For $n = 0, 1, 2, \dots$, let $W_n = \sum_{k=0}^{\lfloor n/3 \rfloor} \binom{2k}{k} \binom{3k}{k} \binom{n}{3k} (-3)^{n-3k}$, where $\lfloor x \rfloor$ is the greatest integer not exceeding x . Then $\{W_n\}$ is an Apéry-like sequence. In this paper we deduce many congruences involving $\{W_n\}$, in particular, we determine $\sum_{k=0}^{p-1} \binom{2k}{k} \frac{W_k}{m^k} \pmod{p}$ for $m = -640332, -5292, -972, -108, -44, -27, -12, 8, 54, 243$ by using binary quadratic forms, where $p > 3$ is a prime. We also prove several congruences for generalized Apéry-like numbers, and pose 29 challenging conjectures on congruences involving binomial coefficients and Apéry-like numbers.

ZHI-HONG SUN
SCHOOL OF MATHEMATICS
AND STATISTICS
HUAIYIN NORMAL UNIVERSITY
HUAIAN, JIANGSU 223300
P. R. CHINA

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