

## Strong arithmetic property of certain Stern polynomials

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**Abstract.** Let  $B_n(t)$  be the  $n$ -th Stern polynomial, i.e., the  $n$ -th term of the sequence defined recursively as  $B_0(t) = 0$ ,  $B_1(t) = 1$  and  $B_{2n}(t) = tB_n(t)$ ,  $B_{2n+1}(t) = B_n(t) + B_{n-1}(t)$  for  $n \in \mathbb{N}$ . It is well known that the  $i$ -th coefficient in the polynomial  $B_n(t)$  counts the number of hyperbinary representations of  $n - 1$  containing exactly  $i$  digits 1. In this note, we investigate the existence of odd solutions of the congruence

$$B_n(t) \equiv 1 + rt \frac{t^{e(n)} - 1}{t - 1} \pmod{m},$$

where  $m \in \mathbb{N}_{\geq 2}$  and  $r \in \{0, \dots, m - 1\}$  are fixed and  $e(n) = \deg B_n(t)$ . We prove that for  $m = 2$  and  $r \in \{0, 1\}$  and for  $m = 3$  and  $r = 0$ , there are infinitely many odd numbers  $n$  satisfying the above congruence. We also present results of some numerical computations.

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