

On k -antichains in the unit n -cube

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Abstract. A *chain* in the unit n -cube is a set $C \subset [0, 1]^n$ such that for every $\mathbf{x} = (x_1, \dots, x_n)$ and $\mathbf{y} = (y_1, \dots, y_n)$ in C , we either have $x_i \leq y_i$ for all $i \in [n]$, or $x_i \geq y_i$ for all $i \in [n]$. We consider subsets A , of the unit n -cube $[0, 1]^n$, that satisfy

$$\text{card}(A \cap C) \leq k, \quad \text{for all chains } C \subset [0, 1]^n,$$

where k is a fixed positive integer. We refer to such a set A as a k -antichain. We show that the $(n - 1)$ -dimensional Hausdorff measure of a k -antichain in $[0, 1]^n$ is at most kn and that the bound is asymptotically sharp. Moreover, we conjecture that there exist k -antichains in $[0, 1]^n$ whose $(n - 1)$ -dimensional Hausdorff measure equals kn , and we verify the validity of this conjecture when $n = 2$.

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