

## Outer inverses, minus partial orders, and triplet invariance

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**Abstract.** In the paper, we obtain an explicit formula for the outer inverses of a regular element in an arbitrary ring. It becomes calculable for outer inverses. We characterize the triplet  $ba^-c$  (resp.  $ba^+c$ ) invariant under all inner inverses  $a^-$  (resp. reflexive inverses  $a^+$ ) of  $a$  in a semiprime ring. It is also proved that if  $R$  is a regular ring and  $a, b, c \in R$ , then the triplet  $b\hat{a}c$  is invariant under all outer inverses  $\hat{a}$  of  $a$  if and only if  $E[a]E[b]E[c] = 0$ . Here, for  $x \in R$ ,  $E[x]$  is the smallest idempotent in the extended centroid of  $R$  such that  $x = E[x]x$ . These answer two questions in Hartwig and Patrício [12].

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