

On the variations of completely multiplicative functions at consecutive arguments

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Abstract. We focus on the class \mathcal{M}_1^* of completely multiplicative functions f whose set of values belong to the unit circle and their related function $\Delta f(n) := f(n+1) - f(n)$. For such functions f , we study the higher iterations $\Delta^m f(n)$ for fixed integers $m \in \{2, 3, \dots, 7\}$, and for each of these we establish an absolute bound for $|\Delta^m f(n)|$. We also characterise those triplets of multiplicative functions f, g, h with unusually small gaps between their consecutive values. All our characterisations and bounds are obtained following new results of O. Klurman and A. P. Mangerel in the context of their proof of an old conjecture of Kátaï characterising subclasses of \mathcal{M}_1^* .

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