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A sharp trigonometric double inequality

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Abstract. We prove that

$$\left(\frac{5-\sqrt{5}}{8}\right)^{\frac{3}{2}}+\frac{1}{2}\sin^3\frac{8\pi}{5}\leqslant\sum_{k=1}^{n}\frac{\sin^3 k\theta}{k}\leqslant 1 \qquad \text{for all integers } n\geqslant 1 \text{ and } \theta\in(0,\pi),$$

where both bounds are sharp. This gives an affirmative answer to a conjecture of Alzer and Koumandos.

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