

On products of consecutive integers

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Abstract. Let p_k be the k -th prime number, and let $\nu_p(n)$ be the p -adic valuation of a positive integer n . Recently, Yang, Luca and Togbé proved that $\nu_p((p_k + 1)(p_k + 2) \cdots (p_{k+1} - 1)) \leq \nu_p((\frac{1}{2}(p_{k+1} - 1))!)$ for any integer $k \geq 5$ and any prime $p \leq \frac{1}{2}(p_{k+1} - 1)$. In this paper, as a corollary, we prove that for any positive real number α , there exists a positive integer K_α such that $\nu_p((p_k + 1)(p_k + 2) \cdots (p_{k+1} - 1)) \leq \nu_p([\alpha(p_{k+1} - 1)]!)$ for any integer $k \geq K_\alpha$ and any prime $p \leq \alpha(p_{k+1} - 1)$.

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