

Three supercongruences for Apéry numbers and Franel numbers

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Abstract. The Apéry numbers A_n and the Franel numbers f_n are defined by

$$A_n = \sum_{k=0}^n \binom{n+k}{2k}^2 \binom{2k}{k}^2 \quad \text{and} \quad f_n = \sum_{k=0}^n \binom{n}{k}^3 \quad (n = 0, 1, \dots).$$

In this paper, we prove three supercongruences for Apéry numbers and Franel numbers conjectured by Z.-W. Sun. For any prime $p \geq 5$,

$$\begin{aligned} \frac{1}{n} \left(\sum_{k=0}^{pn-1} (2k+1)A_k - p \sum_{k=0}^{n-1} (2k+1)A_k \right) &\equiv 0 \pmod{p^{4+3\nu_p(n)}}, \\ \frac{1}{n^3} \left(\sum_{k=0}^{pn-1} (2k+1)^3 A_k - p^3 \sum_{k=0}^{n-1} (2k+1)^3 A_k \right) &\equiv 0 \pmod{p^{6+3\nu_p(n)}}, \end{aligned}$$

and, for any prime p ,

$$\frac{1}{n^3} \left(\sum_{k=0}^{pn-1} (3k+2)(-1)^k f_k - p^2 \sum_{k=0}^{n-1} (3k+2)(-1)^k f_k \right) \equiv 0 \pmod{p^3},$$

where $\nu_p(n)$ denotes the p -adic order of n .

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