

Constrained triangulations, volumes of polytopes, and unit equations

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Abstract. Given a polytope \mathcal{P} in \mathbb{R}^d and a subset U of its vertices, is there a triangulation of \mathcal{P} using d -simplices that all contain U ? We answer this question by proving an equivalent and easy-to-check combinatorial criterion for the facets of \mathcal{P} . Our proof relates triangulations of \mathcal{P} to triangulations of its “shadow”, a projection to a lower-dimensional space determined by U . In particular, we obtain a formula relating the volume of \mathcal{P} with the volume of its shadow. This leads to an exact formula for the volume of a polytope arising in the theory of unit equations.

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