

On cyclic quadrilaterals in euclidean and hyperbolic geometries

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Abstract. Four points ordered in the positive order on the unit circle determine the vertices of a quadrilateral, which is considered either as a euclidean or as a hyperbolic quadrilateral depending on whether the lines connecting the vertices are euclidean or hyperbolic lines. In the case of hyperbolic lines, this type of quadrilaterals are called ideal quadrilaterals. Our main result gives a euclidean counterpart of an earlier result on the hyperbolic distances between the opposite sides of ideal quadrilaterals. The proof is based on computations involving hyperbolic geometry. We also find a new formula for the hyperbolic midpoint of a hyperbolic geodesic segment in the unit disk. As an application of some geometric properties, we provide a euclidean construction of the symmetrization of random four points on the unit circle with respect to a diameter which preserves the absolute cross ratio of quadruples.

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