Surgeries of the Gieseking hyperbolic ideal simplex manifold

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Abstract. In our Novi Sad conference paper (1999), we described Dehn type surgeries of the famous Gieseking (1912) hyperbolic ideal simplex manifold $\mathcal{S}$, leading to compact fundamental domain $\mathcal{S}(k)$, $k = 2, 3, \ldots$ with singularity geodesics of rotation order k, but, as later turned out, with cone angle $2\pi(k-1)/k$. We computed also the volume of $S(k)$, tending to zero if k goes to infinity. That time we naively thought that we obtained orbifolds with the above surprising property. As the reviewer of Math. Rev., Kevin P. Scannell (MR1770996 (2001g:57030)) rightly remarked, "this is in conflict with the well-known theorem of D. A. Kazhdan and G. A. Margulis (1968) and with the work of Thurston, describing the geometric convergence of orbifolds under large Dehn fillings".

In this paper, we update our previous publication. Correctly, we obtained cone manifolds (for $k > 2$), as A. D. Mednykh and V. S. Petrov (2006) kindly pointed out. We complete our discussion and derive the above cone manifold series (Gies.1 and Gies.2) in two geometrically equivalent form, by the half turn symmetry of any ideal simplex. Moreover, we obtain a second orbifold series (Gies.3 and Gies.4), tending to the regular ideal simplex as the original Gieseking manifold.

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