

## On additive functions with additional derivation properties

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**Abstract.** The purpose of this paper is to introduce the notion of a generalized derivation which derivates a prescribed family of smooth vector-valued functions of several variables. The basic calculus rules are established and then a result derived which shows that if an additive function  $d$  is a derivation with respect to a differentiable function  $f$  which satisfies an addition theorem, then  $d$  is also a derivation with respect to the determining operation. As an application of this approach, a new proof of a generalization of a recent result of Maksa is obtained. We also extend the result of Nishiyama and Horinouchi and formulate two open problems.

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