D'Alembert's functional equation on monoids with both an endomorphism and an anti-endomorphism

By MOHAMED AYOUBI (Meknes), DRISS ZEGLAMI (Meknes)
and AZIZ MOUZOUN (Meknes)

Abstract. Let M be a topological monoid, e its neutral element, let $\mathbb{K}$ denote a division ring of characteristic $\neq 2$, and let $\psi : M \to M$ (resp. $\varphi : M \to M$) be a continuous anti-endomorphism (resp. endomorphism) of M as a semigroup. We solve the functional equation

$$g(xy) - g(x\psi(y)) = cg(x)g(y), \quad x, y \in M,$$

where $g : M \to \mathbb{K}$ is the unknown function and $c \in \mathbb{K}\backslash\{0\}$. This enables us to find, when $\psi(e) = \varphi(e) = e$, the solutions $g : M \to \mathbb{C}$ of each of the new functional equations

$$g(\varphi(x)y) \pm g(x\psi(y)) = 2g(x)g(y), \quad x, y \in M.$$

We also find the continuous, complex valued solutions of a Van Vleck's functional equation with an anti-endomorphism on M.

MOHAMED AYOUBI &
DRISS ZEGLAMI &
AZIZ MOUZOUN
DEPARTMENT OF MATHEMATICS
E.N.S.A.M, MOULAY ISMAIL UNIVERSITY
B.P. 15290 AL MANSOUR, MEKNES
MOROCCO