

Some functional lower bounds for Fejér's sine polynomial

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Abstract. Let $\alpha \in \mathbb{R}$. We prove that

$$\begin{aligned} \sum_{k=1}^n \frac{\sin k\theta}{k} \geq & \left(\frac{335}{576} + \frac{31}{128}\alpha \right) \sin \theta + \left(\frac{39}{128} + \frac{5}{32}\alpha \right) \sin 2\theta + \left(\frac{113}{2304} + \frac{21}{256}\alpha \right) \sin 3\theta \\ & + \left(\frac{25}{256} - \frac{5}{64}\alpha \right) \sin 4\theta + \left(\frac{125}{2304} - \frac{25}{256}\alpha \right) \sin 5\theta > \theta^2 \left(\cot \frac{\theta}{2} - \frac{\pi - \theta}{2} \right) \end{aligned}$$

for every integer $n \geq 2$ and $\theta \in (0, \pi)$ if and only if

$$\frac{\pi^2}{24} \leq \alpha \leq \alpha_0,$$

where α_0 denotes the unique real zero of the cubic polynomial

$$41721661440t^3 - 57761574336t^2 + 46230194016t - 13479325596.$$

Both bounds for α are sharp. This sharpens a result due to Alzer and Koumandos.

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