

Correction to my paper "The quasi-series decomposition of two-terminal graphs"

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1. Dr. G. POLLÁK has kindly called my attention that the discussion of Case 4/b/δ of the proof of Theorem 1 in my cited paper is incomplete. In order to correct the proof, I shall now point out a new lemma and give a more detailed treatment of the final part of the mentioned proof.

2. Beside the lemma exposed in § 4 of [1], we need also

Lemma 11. *Let $\mathfrak{G}$ be an indecomposable graph having at least three final edges. Let k_1 and $k_2(BQ)$ be two final edges of $\mathfrak{G}$; assume that B is an inner vertex of each 2-subgraph of $\mathfrak{G}$. Then there exists a chain $a(PB)$ in $\mathfrak{G}$ such that a contains k_1 but it does not contain k_2 .*

PROOF. If we delete k_2 in $\mathfrak{G}$ (for a moment), then the resulting 2-graph is likewise indecomposable. Hence $\mathfrak{G}$ has two disjoint paths b, c which do not contain k_2 . Furthermore, there exists a (possibly degenerated) chain $d(CB)$ such that

d contains no terminal of $\mathfrak{G}$,

C is an inner vertex in b or in c , and

d has no vertex, different from C , which occurs in b or in c .

The symmetry makes possible to suppose that C is contained in c .

Case 1: k_1 is contained in b or c . Then the chain

$$b \cdot c^{-1}[QC] \cdot d \quad .$$

satisfies the conclusion of the lemma.

Case 2: neither b nor c contains k_1 . Let e be a path containing k_1 . Denote by E the last vertex of e which differs from Q and is contained in one of b, c, d . We can distinguish five possibilities according to the situation of E :

$E = P$,

E is an inner vertex of b ,

E is an inner vertex of $c[PC]$,

E is a vertex of $c[CQ]$,

E is a vertex of d .

According to the five possibilities enumerated, one of the chains

$$\begin{aligned} & e \cdot c^{-1}[QC] \cdot d, \\ & b[PE] \cdot e[EQ] \cdot c^{-1}[QC] \cdot d, \\ & c[PE] \cdot e[EQ] \cdot c^{-1}[QC] \cdot d, \\ & b \cdot e^{-1}[QE] \cdot c^{-1}[EC] \cdot d, \\ & b \cdot e^{-1}[QE] \cdot d[EB] \end{aligned}$$

fulfils the conclusion, respectively.

3. On p. 104 of [1], the final section of the proof of Case 4/b/ δ (beginning with the words "There are two alternatives") should be replaced by what follows:

Denote by $\mathfrak{H}_1$ and $\mathfrak{H}_2$ the narrowest (non-trivial) 2-subgraphs of $\mathfrak{G}^*$ which contain k_1 and k_2 , respectively. It suffices to study only the case $\mathfrak{H}_1 \supseteq \mathfrak{H}_2$. Denote by $\mathfrak{H}^*$ the widest proper 2-subgraph of $\mathfrak{H}_1$ and by $\mathfrak{R}$ the narrowest 2-subgraph satisfying $\mathfrak{H}_1 \supseteq \mathfrak{R} \supseteq \mathfrak{H}_2$ (provided that such a $\mathfrak{H}^*$ or $\mathfrak{R}$ does exist). The graphs $\mathfrak{H}_1/\mathfrak{H}^*$ and $\mathfrak{R}/\mathfrak{H}_2$ are irreducible or they consist of two edges. The subsequent seven situations are possible:

- (i) $\mathfrak{H}_1 = \mathfrak{H}_2$,
- (ii) $\mathfrak{H}_1/\mathfrak{H}^*$ (exists and) consists of two parallel-composed edges, moreover $\mathfrak{H}_2 \subseteq \mathfrak{H}^*$,
- (iii) $\mathfrak{H}_1/\mathfrak{H}^*$ is irreducible, $\mathfrak{H}_2 = \mathfrak{H}^*$, k_2 is a series component of $\mathfrak{H}_2$,
- (iv) $\mathfrak{H}_1/\mathfrak{H}^*$ is irreducible, $\mathfrak{H}_2 = \mathfrak{H}^*$, $\mathfrak{H}_2$ is indecomposable,
- (v) $\mathfrak{H}_1/\mathfrak{H}^*$ is irreducible, $\mathfrak{H}_2 \subset \mathfrak{H}^*$, and $\mathfrak{R}/\mathfrak{H}_2$ consists of two parallel-composed edges,
- (vi) $\mathfrak{H}_1/\mathfrak{H}^*$ is irreducible, $\mathfrak{H}_2 \subset \mathfrak{H}^*$, and $\mathfrak{R}/\mathfrak{H}_2$ consists of two series-composed edges,
- (vii) $\mathfrak{H}_1/\mathfrak{H}^*$ and $\mathfrak{R}/\mathfrak{H}_2$ are irreducible, $\mathfrak{H}_2 \subset \mathfrak{H}^*$.

In each of these cases we are going to point out that there exists a chain in $\mathfrak{G}^*$ between P and A such that both k_1 and k_2 occur in it.

If (i) is valid, then k_1 and k_2 form a separating pair in $\mathfrak{H}_1$; we conclude by Lemma 4. If (ii) holds, then there exists a chain in $\mathfrak{H}_1$ between Q_2 and A such that this chain contains k_1 and k_2 (Lemma 10); it can be completed by a suitable path of $\mathfrak{G}^*$ to a chain connecting P and A . (We have utilized Lemma 5, too; this result must be kept in mind also in what follows.) In cases (iii)–(vii) k_1 and h^* are final edges in $\mathfrak{H}_1/\mathfrak{H}^*$. If (iii) is true, then it suffices to consider the possibility when k_1, k_2 do not form a separating pair in $\mathfrak{H}_1$; $\mathfrak{H}_1/\mathfrak{H}_2$ has at least three final edges, thus we can apply Lemma 11 in $\mathfrak{H}_1/\mathfrak{H}_2$ for k_1 and h_2 . Among the remaining four possibilities, if (v) is valid, then Lemma 8 is applicable in $\mathfrak{H}_2$; if one of (iv), (vi), (vii) holds, we can utilize Lemma 10.

Reference

- [1] A. ÁDÁM, The quasi-series decomposition of two-terminal graphs, Publ. Math. Debrecen, 10 (1963), 96–107.

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