Publ. Math. Debrecen
39 / 3–4 (1991), 243–251

Finsler connection transformations associated to a general Finsler metrical structure of Miron type (II)

By PETRE STAVRE (Craiova) and FRANCISC C. KLEPP (Timişoara)

1. Introduction

Let M be an n-dimensional differentiable manifold of class C^∞, (TM, π, M) its tangent bundle, (x^i, y^i) the local coordinates of a point $u \in TM$ in a local map, and let $\{\delta/\delta x^i, \partial/\partial y^i\}$ be an adapted basis, where $\delta/\delta x^i = \partial/\partial x^i - N_i^k \partial/\partial y^k$, N_k^i being the coefficients of a fixed non-linear connection N. Let $\mathcal{T}$ be the general group of Finsler connection transformations [6]: $t : F\Gamma(N) = (N_j^i, F_{jk}^i, C_{jk}^i) \to F\bar{\Gamma}(\bar{N}) = (\bar{N}_j^i, \bar{F}_{jk}^i, \bar{C}_{jk}^i)$ of the form:

$$(1.1)\quad \bar{N}_j^i(x,y) = N_j^i - A_j^i;\ \bar{F}_{jk}^i = F_{jk}^i + C_{jk}^a A_a^i + B_{jk}^i;\ \bar{C}_{jk}^i = C_{jk}^i + D_{jk}^i$$

where $A \in Z_1^1(M)$ and $B, D \in Z_2^1(M)$ are arbitrary Finsler tensors; and $\mathcal{T}_N = \{t \in \mathcal{T}, t = t(0, B, D)\}$ the subgroup of $\mathcal{T}$ formed by transformations $t : (N, F, C) \to (N, \bar{F}, \bar{C})$, which preserve the non-linear connection N. In a previous paper [12] we have introduced a classification method for the Finsler connection transformations $t \in \mathcal{T}_N$, using a metrical Finsler structure of Miron type $g = (g_{ij})$. In the present paper a generalization of this method for the case of the general group $\mathcal{T}$ of the Finsler connection transformations is given.

The notions and notations of M. Matsumoto [5] and R. Miron [6] are used.

2. t_g and $t_{\text{non}\,g}$ transformations

Definition 2.1. A Finsler connection transformation $t \in \mathcal{T}$ is called a t_g transformation if and only if we have:

$$(2.1)\qquad g_{aj\|k} - \bar{N}_k^s g_{aj}\big\|_s = g_{aj|k} - N_k^s g_{aj}\big|_s\ ; \quad g_{aj}\big\|_k = g_{aj}\big|_k$$

where $|, \big|$ and $\|, \big\|$ denote the h- and v-covariant derivatives relative to (N, F, C) and $(\bar{N}, \bar{F}, \bar{C})$ respectively.

Definition 2.2. A Finsler connection transfromation $t \in \mathcal{T}$ is called a $t_{\mathrm{non}\, g}$ transformation if and only if at least one of the conditions (2.1) is not satisfied.

If $\bar{N} = N$, then $t \in \mathcal{T}_N$ and we obtain the Definitions 2.1 and 2.2 from [12].

We denote:

$$G^i_{jk} = \frac{1}{2} g^{ir}(g_{rj|k} + g_{rk|j} - g_{jk|r}) \tag{2.2}$$

$$H^i_{jk} = \frac{1}{2} g^{ir}(g_{rj}\big|_k + g_{rk}\big|_j - g_{jk}\big|_r) \tag{2.3}$$

$$N^i_{jk} = \frac{1}{2} g^{ir}(g_{rj}\big|_a N^a_k + g_{rk}\big|_a N^a_j - g_{jk}\big|_a N^a_r) \tag{2.4}$$

and analogously $\bar{G}^i_{jk}, \bar{H}^i_{jk}$ and $\bar{N}^i_{jk}$ for $F\bar{\Gamma}(\bar{N})$.

From (2.1), (2.2), (2.3) and (2.4) follow the invariants:

$$\bar{G}^i_{jk} + \bar{N}^i_{jk} = G^i_{jk} + N^i_{jk};\ \bar{H}^i_{jk} = H^i_{jk}. \tag{2.5}$$

Thus we have:

Theorem 2.1. *If $t \in \mathcal{T}$ is a t_g-transformation then (2.5) are invariants of this transformation.*

Example 2.1. Any $t(A, 0, 0) \in \mathcal{T}$ transformation i.e. (1.1) transformation with $B = D = 0$, is a t_g-transformation.

If we denote:

$$\overset{*}{U}{}^i_{jk} = \frac{1}{2}(U^i_{jk} + g_{sj} g^{ir} U^s_{kr} + g_{sk} g^{ir} U^s_{jk});\quad U^i_{jk} \in Z^1_2(M) \tag{2.6}$$

then, using the method from [12], from (1.1), (2.2) and (2.3) it follows the:

Theorem 2.2. *A Finsler connection transformation $t \in \mathcal{T}$ is a t_g-transformation if and only if it is of the form:*

$$\begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \\ \bar{F}^i_{jk} = F^i_{jk} + C^i_{ja} A^a_k + \overset{*}{\bar{T}}{}^i_{jk} - \overset{*}{T}{}^i_{jk} - \overset{*}{\Theta}{}^i_{jk} \\ \bar{C}^i_{jk} = C^i_{jk} + \overset{*}{\bar{S}}{}^i_{jk} - \overset{*}{S}{}^i_{jk} \end{cases} \tag{2.7}$$

where $\overset{*}{\bar{T}}$ is equal to $\overset{*}{U}$ if $U = \bar{T}$; $\overset{*}{T}$ is equal to $\overset{*}{U}$ if $U = T$; $\overset{*}{\Theta}$ is equal to $\overset{*}{U}$ if $U^i_{jk} = C^i_{ja} A^a_k - C^i_{ka} A^a_j$; $\overset{*}{\bar{S}}$ is equal to $\overset{*}{U}$ if $U = \bar{S}$ and $\overset{*}{S}$ is equal to $\overset{*}{U}$ if

$U = S$; $A^a_j \in Z^1_1(M)$ is an arbitrary Finsler tensor, T, S and $\bar{T}, \bar{S}$ are the h- and v-torsions of $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$ respectively.

Theorem 2.3. *The set of all Finsler connections $F\bar{\Gamma}(\bar{N})$ obtained from a fixed Finsler connection $F\Gamma(N)$ by a t_g-transformation is given by (2.7), where $A^a_k \in Z^1_1(M)$ is an arbitrary Finsler tensor; $\bar{T}, \bar{S}$ are arbitrary skewsymmetric tensors: $\bar{T}^i_{jk} = -\bar{T}^i_{kj}$; $\bar{S}^i_{jk} = -\bar{S}^i_{kj}$ and T, S are the h- and v-torsions of the fixed Finsler connection $F\Gamma(N)$.*

It follows that the transformations (2.7) are characterized by the invariants (2.1) or (2.5).

Corollary 2.1. *If $A^i_j = 0$, then $\bar{N} = N$ and we obtain the g transformations $t_g \in \mathcal{T}_N$ from [12].*

We denote:

$$A^i_{jk} = \frac{1}{2} g^{ia} (g_{aj}\big|_b A^b_k + g_{ak}\big|_b A^b_j - g_{jk}\big|_b A^b_k) \tag{2.8}$$

If $t \in \mathcal{T}$ is a $t_{\text{non}\, g}$-transformation, then using the method from [12] it follows from (1.1) the:

Theorem 2.4. *A Finsler connection transformation $t \in \mathcal{T}$ is a $t_{\text{non}\, g}$-transformation if and only if it is of the form:*

$$\begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \\ \bar{F}^i_{jk} = F^i_{jk} + C^i_{ja} A^a_k + \overset{*}{\bar{T}}{}^i_{jk} - \overset{*}{T}{}^i_{jk} - \overset{*}{\Theta}{}^i_{jk} + G^i_{jk} - \bar{G}^i_{jk} + A^i_{jk} \\ \bar{C}^i_{jk} = C^i_{jk} + \overset{*}{\bar{S}}{}^i_{jk} - \overset{*}{S}{}^i_{jk} + H^i_{jk} - \bar{H}^i_{jk} \end{cases} \tag{2.9}$$

and:

Theorem 2.5. *Let $F\Gamma(N)$ be a fixed Finsler connection with the h-torsion T and v-torsion S. Then any other Finsler connection $F\bar{\Gamma}(\bar{N})$ which does not possess the property (2.1) is given by (2.9), where $\bar{T}, \bar{S}$ are arbitrary skewsymmetric Finsler tensors; $\bar{G}, \bar{H}$ are arbitrary symmetric Finsler tensors: $\bar{G}^i_{jk} = \bar{G}^i_{kj}$; $\bar{H}^i_{jk} = \bar{H}^i_{kj}$; $A^i_j \in Z^1_1(M)$ is an arbitrary Finsler tensor and $\bar{H} \neq H$ or $G^i_{jk} - \bar{G}^i_{jk} + A^i_{jk} \neq 0$.*

If we fix $\bar{T}, \bar{S}, \bar{G}, \bar{H}$ and A^i_j, then we obtain a Finsler connection $F\bar{\Gamma}(\bar{N})$ which has the h-torsion $\bar{T}$, the v-torsion $\bar{S}$ and $\bar{G}, \bar{H}$ satisfies the relations:

$$\begin{aligned} \bar{G}^i_{jk} &= \frac{1}{2} g^{ir} (g_{rj\|k} + g_{rk\|j} - g_{jk\|r}); \\ \bar{H}^i_{jk} &= \frac{1}{2} g^{ir} (g_{rj}\big\|_k + g_{rk}\big\|_j - g_{jk}\big\|_r) \end{aligned} \tag{2.10}$$

From (2.9) follow also special cases of $t_{\text{non}-g}$-transformations.

We obtain the:

Theorem 2.6. (*Separation Theorem*). *Any Finsler connection transformation $t \in \mathcal{T}$ is a t_g-transformation ($t \in \mathcal{T}_g$), or a $t_{\text{non } g}$-transformation ($t \in \mathcal{T}_{\text{non } g}$) and we have: $\mathcal{T} = \mathcal{T}_g \cup \mathcal{T}_{\text{non } g}$; $\mathcal{T}_g \cap \mathcal{T}_{\text{non } g} = \emptyset$.*

If in (2.9) we give up one, or both of the conditions $\bar{H} \neq H$ and $G^i_{jk} - \bar{G}^i_{jk} + A^i_{jk} = 0$, then we obtain the:

Theorem 2.7. (*General Theorem*). *Any Finsler connection transformation $t \in \mathcal{T}$ is of the form (2.9).*

This Theorem is important, since it is expressed solely in the terms of the two Finsler connections $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$ and thus we have the direct interpretation of the terms $\bar{T}, T$; $\bar{S}, S$; $\bar{G}, G$; A, for every definiteness of the connections. Consequently the invariants of these transformations can be studied elegantly in the special cases.

Corollary 2.3. *Any Finsler connection transformation $t \in \mathcal{T}_N$ is of the form:*

$$\tag{2.11} \begin{aligned} &\bar{N}^i_j = N^i_j; \quad \bar{F}^i_{jk} = F^i_{jk} + \overset{*}{\bar{T}}{}^i_{jk} - \overset{*}{T}{}^i_{jk} + G^i_{jk} - \bar{G}^i_{jk}; \\ &\bar{C}^i_{jk} = C^i_{jk} + \overset{*}{\bar{S}}{}^i_{jk} - \overset{*}{S}{}^i_{jk} + H^i_{jk} - \overset{*}{H}{}^i_{jk} \end{aligned}$$

In this way the result of [12] is obtained.

3. Special classes of $t \in \mathcal{T}$ transformations

In the theory of linear connections the Schouten tensor play a prominent part. In the theory of Finsler connections we can define two Finsler-Schouten tensors:

$$\tag{3.1} \underset{1}{I}{}^i_{jk} = T^i_{jk} - \frac{1}{n-1}(\delta^i_j T_k - \delta^i_k T_j); \ \underset{2}{I}{}^i_{jk} = S^i_{jk} - \frac{1}{n-1}(\delta^i_j S_k - \delta^i_k S_j)$$

where $T_k = T^i_{ik}$, $S_k = S^i_{ik}$ and analogously two tensors $\underset{1}{\bar{I}}$ and $\underset{2}{\bar{I}}$ for $F\bar{\Gamma}$. Since we have:

$$\tag{3.2} \overset{*}{\bar{T}}{}^i_{jk} = \overset{*}{T}{}^i_{jk} = \alpha_j \delta^i_k - g_{jk}\alpha^i; \quad \overset{*}{\bar{S}}{}^i_{jk} = \overset{*}{S}{}^i_{jk} = \beta_j \delta^i_k - g_{jk}\beta^i$$

where $\alpha_j = \frac{1}{n-1}(T_j - \bar{T}_j)$; $\beta_j = \frac{1}{n-1}(S_j - \bar{S}_j)$, in case of $\underset{1}{\bar{I}} = \underset{1}{I}$, $\underset{2}{\bar{I}} = \underset{2}{I}$ we arrive to the following theorems:

Theorem 3.1. *Any t_g-transformation, which admits the invariants $\bar{\underset{1}{I}} = \underset{1}{I}$ and $\bar{\underset{2}{I}} = \underset{2}{I}$ is of the form:*

$$(3.3) \qquad \begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \quad \bar{F}^i_{jk} = F^i_{jk} + C^i_{jr}A^r_k - \overset{*}{\Theta}{}^i_{jk} + \alpha_j\delta^i_k - g_{jk}\alpha^i \\ \bar{C}^i_{jk} = C^i_{jk} + \beta_j\delta^i_k - g_{jk}\beta^i \end{cases}$$

Theorem 3.2. *Any $t_{\text{non}\, g}$-transformation, which admits the invariants $\bar{\underset{1}{I}} = \underset{1}{I}$ and $\bar{\underset{2}{I}} = \underset{2}{I}$ is of the form:*

$$(3.4) \qquad \begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \\ \bar{F}^i_{jk} = F^i_{jk} + C^i_{jr}A^r_k - \overset{*}{\Theta}{}^i_{jk} + G^i_{jk} - \bar{G}^i_{jk} + A^i_{jk} + \alpha_j\delta^i_k - g_{jk}\alpha^i \\ \bar{C}^i_{jk} = C^i_{jk} + H^i_{jk} - \bar{H}^i_{jk} + \beta_j\delta^i_k - g_{jk}\beta^i \end{cases}$$

If $A^i_j = 0$, then $\bar{N} = N$ and we obtain the corresponding theorems from [12].

Particularly we obtain:

Theorem 3.3. *Any t_g-transformation of semisymmetric ($\underset{1}{I} = 0$, $\underset{2}{I} = 0$) Finsler connections is of the form (3.3).*

Theorem 3.4. *Any $t_{\text{non}\, g}$-transformation of semisymmetric Finsler connections is of the form (3.4).*

If $F\Gamma(N)$ is a metrical Finsler connection, then $g_{ij|k} = o$, $g_{ij}\big|_k = o$ and it follows:

Theorem 3.5. *Any transformation $t \in \mathcal{T}$ of metrical Finsler connections is a t_g-transformation.*

Theorem 3.6. *Any transformation $t \in \mathcal{T}$ which transforms a non-metrical Finsler connection $F\Gamma(N)$ in a metrical Finsler connection $F\bar{\Gamma}(\bar{N})$ is a $t_{\text{non}\, g}$-transformation of the form:*

$$(3.5) \qquad \begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \\ \bar{F}^i_{jk} = F^i_{jk} + C^i_{jr}A^r_k + \bar{\overset{*}{T}}{}^i_{jk} - \overset{*}{T}{}^i_{jk} - \overset{*}{\Theta}{}^i_{jk} + G^i_{jk} + A^i_{jk} \\ \bar{C}^i_{jk} = C^i_{jk} + \bar{\overset{*}{S}}{}^i_{jk} - \overset{*}{S}{}^i_{jk} + H^i_{jk} \end{cases}$$

This transformation solves the metrization problem of KAWAGUCHI. From the Theorems 3.1 and 3.5 follow the results of [9], [10].

Obviously any other conformal transformation compatible with a complex structure is obtained from (2.7) or (2.9) by particularization.

In [6], [7], [8] R. MIRON establishes the following metrizations:

Theorem. *If $F\Gamma(N) = (N, F, C)$ is a fixed non-metrical Finsler connection, then any other metrical Finsler connection is given by:*

$$(3.6)\qquad \begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \\ \bar{F}^i_{jk} = F^i_{jk} + C^i_{jr}A^r_k + \frac{1}{2}g^{ir}(g_{rj|k} + g_{kj}\big|_a A^a_k) + \Omega^{ir}_{sj}X^s_{rk} \\ \bar{C}^i_{jk} = C^i_{jk} + \frac{1}{2}g^{ir}g_{rj}\big|_k + \Omega^{ir}_{sj}Y^s_{rk} \end{cases}$$

where $X, Y \in Z^1_2(M)$ are arbitrary Finsler tensor fields and $\Omega^{ir}_{sj} = \frac{1}{2}(\delta^i_s\delta^r_j - g_{sj}g^{ir})$ is the Obata operator $F\Gamma(N)$.

The transformations (3.6) are very important, since for any definiteness of $X, Y \in Z^1_1(M)$ a class of metrical Finsler connections $F\bar{\Gamma}(\bar{N})$ obtained from a non-metrical Finsler connection $F\Gamma(N)$ follows. It is an elegant generalization of the Kawaguchi metrizations method. By definiteness of $A^i_j \in Z^1_1(M)$ we obtain actually the metrical connection $F\bar{\Gamma}(\bar{N})$.

We shall solve now the inverse problem: Let $F\Gamma(N)$ be a fixed non-metrical Finsler connection and $F\bar{\Gamma}(\bar{N})$ an another given metrical Finsler connection. Which relations hold between $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$? What form has the transformation $t : F\Gamma(N) \to F\bar{\Gamma}(\bar{N})$?

Between $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$ obviously exist the relations (3.6), but in what extent are X and Y arbitrary? This is the problem, which has to be solved. From (3.6) it follows:

$$(3.7)\qquad \begin{aligned} \bar{T}^i_{jk} =& T^i_{jk} + C^i_{jr}A^r_k - C^i_{kr}A^r_k + \frac{1}{2}g^{ir}(g_{rj|k} - g_{rk|j}) + \\ &+ \frac{1}{2}g^{ir}(g_{rj}\big|_a A^a_k - g_{rk}\big|_a A^a_j) + \Omega^{ir}_{sj}X^s_{rk} - \Omega^{ir}_{sk}X^s_{rj} \end{aligned}$$

$$(3.8)\qquad \bar{S}^i_{jk} = S^i_{jk} + \frac{1}{2}g^{ir}(g_{rj}\big|_k - g_{rk}\big|_j) + \Omega^{ir}_{sj}Y^s_{rk} - \Omega^{ir}_{sk}Y^s_{rj}$$

Consequently we obtain a system of tensor equations of the form:

$$(3.9)\qquad \Omega^{ir}_{sj}X^s_{rk} - \Omega^{ir}_{sk}X^s_{rj} = \underset{1}{\sigma}{}^i_{jk}$$

$$(3.10)\qquad \Omega^{ir}_{sj}Y^s_{rk} - \Omega^{ir}_{sk}Y^s_{rj} = \underset{2}{\sigma}{}^i_{jk}$$

where:

$$(3.11)\qquad \begin{aligned} \underset{1}{\sigma}{}^i_{jk} =& (\bar{T}^i_{jk} - T^i_{jk}) - (C^i_{jr}A^r_k - C^i_{kr}A^r_j) - \\ &- \frac{1}{2}g^{ir}(g_{rj|k} - g_{rk|j}) - \frac{1}{2}g^{ir}(g_{rj}\big|_a A^a_k - g_{rk}\big|_a A^a_j) \end{aligned}$$

$$\underset{2}{\sigma}{}^i_{jk} = (\bar{S}^i_{jk} - S^i_{jk}) - \frac{1}{2}g^{ir}(g_{rj}\big|_k - g_{rk}\big|_j) \tag{3.12}$$

The general solution of a systems of the form (3.9)–(3.10) is given in [13] using the theory of fixed points. Here we give a direct solution by comparison of (3.6) and (3.5). It follows that a general solution is of the form:

$$\begin{aligned} \Omega^{ir}_{sj}X^s_{rk} = \overset{*}{\bar{T}}{}^i_{jk} - \overset{*}{T}{}^i_{jk} - \overset{*}{\Theta}{}^i_{jk} + \frac{1}{2}g^{ir}(g_{rk|j} - g_{jk|r}) + \\ + \frac{1}{2}g^{ir}(g_{rk}\big|_a A^a_j - g_{jk}\big|_a A^a_r) \end{aligned} \tag{3.13}$$

$$\Omega^{ir}_{sj}Y^s_{rk} = \bar{S}^i_{jk} - S^i_{jk} + \frac{1}{2}g^{ir}(g_{rk}\big|_j - g_{jk}\big|_r) \tag{3.14}$$

Denoting the right-hand member of (3.13) and (3.14) by B^i_{jk} and D^i_{jk} respectively, we have:

$$\overset{*}{\Omega}{}^{ir}_{sj}B^s_{rk} = 0; \quad \overset{*}{\Omega}{}^{ir}_{sj}D^s_{rk} = 0 \tag{3.15}$$

where $\overset{*}{\Omega}{}^{ir}_{sj} = \frac{1}{2}(\delta^i_s\delta^r_j + g^{ir}g_{sj})$ is the Obata operator.

From (3.15) it follows the compatibility of the system (3.13)–(3.14) and it has the solution:

$$X^i_{jk} = B^i_{jk} + \overset{*}{\Omega}{}^{is}_{rj}U^r_{sk} \tag{3.16}$$

$$Y^i_{jk} = D^i_{jk} + \overset{*}{\Omega}{}^{is}_{rj}V^r_{sk} \tag{3.17}$$

where $U, V \in Z^1_2(M)$ are arbitrary and B^i_{jk}, D^i_{jk} are given by the right-hand members of (3.13) and (3.14) respectively.

Consequently we have the:

Theorem 3.7. *Between a non-metrical and a metrical Finsler connections $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$, there exists the relation (3.6), where X, Y are arbitrary up to a transformation of form (3.16)–(3.17).*

Consequently X, Y are not completly arbitrary. They must be choosen of the form (3.16)–(3.17) in order to satisfy the equations (3.6).

In case when $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$ are both metrical we obtain from [1] and [8] the results:

$$\begin{cases} \bar{N}^i_j = N^i_j - A^i_j; \quad \bar{F}^i_{jk} = F^i_{jk} + C^i_{ja}A^a_k + \Omega^{ir}_{sj}X^s_{rk} \\ \bar{C}^i_{jk} = C^i_{jk} + \Omega^{ir}_{sj}Y^s_{rk} \end{cases} \tag{3.18}$$

From (3.16) and (3.17) follows the general solution:

$$X^i_{jk} = \overset{*}{\bar{T}}{}^i_{jk} - \overset{*}{T}{}^i_{jk} - \overset{*}{\Theta}{}^i_{jk} + \overset{*}{\Omega}{}^{ir}_{sj} U^s_{rk} \tag{3.19}$$

$$Y^i_{jk} = \overset{*}{\bar{S}}{}^i_{jk} - \overset{*}{S}{}^i_{jk} + \overset{*}{\Omega}{}^{ir}_{sj} V^s_{rk} \tag{3.20}$$

where $U, V \in Z^1_2(M)$. Thus we have:

Theorem 3.8. *Between two given metrical Finsler connections* $F\Gamma(N)$ *and* $F\bar{\Gamma}(\bar{N})$ *the relations* (3.18) *exist, where* X *and* Y *are given by* (3.19) *and* (3.20).

Consequently neither in this case are X, Y completly arbitrary, only up to a transformation (3.19)–(3.20).

Summary

1. The Miron-Hashiguchi transformations are used if we wish to determine all metrical Finsler connections $F\bar{\Gamma}(\bar{N})$ from a fixed Finsler connection $F\Gamma(N)$.

2. The transformations (3.5) (or equivalently (3.6), (3.16), (3.17)) are used if we wish to determine metrical Finsler connections $F\bar{\Gamma}(\bar{N})$ with special properties (for example: $\underset{1}{\bar{I}} = \underset{1}{I}$, $\underset{2}{\bar{I}} = \underset{2}{I}$ or $\underset{1}{\bar{I}} = 0$, $\underset{2}{\bar{I}} = 0$) starting from a fixed non-metrical Finsler connection $F\Gamma(N)$.

3. The transformations (3.18) are used if we wish to determine arbitrary metrical Finsler connections $F\bar{\Gamma}(\bar{N})$, starting from a fixed metrical Finsler connection $F\Gamma(N)$.

4. The transformations (2.7) are used if we wish to determine Finsler connections with the properties (2.1) (particularly also metrical Finsler connections) whose torsions $\bar{T}, \bar{S}$ have given properties (for example: $\underset{1}{\bar{I}} = \underset{1}{I}$, $\underset{2}{\bar{I}} = \underset{2}{I}$ or $\underset{1}{\bar{I}} = 0$, $\underset{2}{\bar{I}} = 0$, and so on).

If $F\Gamma(N)$ and $F\bar{\Gamma}(\bar{N})$ are metrical we can start from (3.18), (3.19), (3.20).

5. The Finsler connection transformations $t : F\Gamma(N) \to F\bar{\Gamma}(\bar{N})$ compatible with a metrical structure or with other structures (complex, symplectic, etc.) studied by G. Atanasiu [1], [2], [3] can be obtained explicitely from (2.7) by imposing of the corresponding conditions.

6. The conformal Finsler connection transformations $t : F\Gamma(N, \omega) \to F\bar{\Gamma}(\bar{N}, \bar{\omega})$ [4] are obtained from (2.9) in an explicit form, without the indetermination of ΩX and ΩY.

7. In the general case, when *we have no* information relative to the relations between these connections and g, then the general transformations from the Theorem 2.7 are used, imposing the desired conditions.

In this way a *synthesis* is obtained in the study or Finsler connection transformations.

We wish to express our thanks Prof. Dr. doc. RADU MIRON for the help given in approching the theory of this transformations.

References

[1] GH. ATANASIU, Structures et connexions Finsler presque complexe et presque Hermitiennes, *Proc. Univ. Braşov* (1980), 41–53.

[2] GH. ATANASIU, Variétés differentiables doués de couples Finsler, *Proc. Univ. Braşov* (1982), 35–67; 69–73.

[3] GH. ATANASIU, B. SINHA and S. K. SERIGH, Almost contact metrical structures and connections, *Proc. Univ. Braşov* (1984), 29–36.

[4] I. GHINEA, Conformal Finsler connections, *Proc. Univ. Braşov* (1980), 55–68.

[5] M. MATSUMOTO, Foundations of Finsler geometry and special Finsler spaces, *Kaiseisha Press, Japan* (1986).

[6] R. MIRON, Introduction to the theory of Finsler spaces, *Proc. Nat. Sem. on Finsler Spaces Braşov* **1** (1980), 131–184.

[7] R. MIRON and M. HASHIGUCHI, Metrical Finsler connections, *Rep. Fac. Sci. Kagoshima University No.* **12** (1979), 21–35.

[8] R. MIRON, Metrical Finsler structures and metrical Finsler connections, *J. Math. Kyoto Univ.* **23–2** (1983), 219–214.

[9] P. STAVRE, Metrical Finsler connections $TI(g)$, *An. Univ. "Al. I. Cuza" Iaşi* **4** (1984), 87–90.

[10] P. STAVRE and FR. KLEPP, General Finsler connection transformations with invariant I^i_{jk}-tensors, *Proc. Romanian-Japanese Colloquium, Braşov* (1984), 165–170.

[11] P. STAVRE, On the transformations $t \in \mathcal{T}_{NI}, t \in \mathcal{T}_{NJ}$, *Proc. Nat. Sem. on Finsler Spaces, Braşov* **3** (1984), 153–166.

[12] P. STAVRE and FR. KLEPP, Finsler connection transformations associated to a general Finsler metrical strucrure of R. Miron I, *Publicationes Math., Debrecen* **35** (1988), 103–113.

[13] P. STAVRE and FR. KLEPP, f_Ω-mapping with fixed points, *Proc. of the Symp. of Math. Timişoara* (1986), 145–148.

PETRE STAVRE
UNIVERSITY OF CRAIOVA
R-1100 CRAIOVA
BLD. A.I. CUZA 13

FRANCISC C. KLEPP
"TRAIAN VUIA" POLYTECHNIC INSTITUTE
R-1900 TIMIŞOARA
BLD. 30 DECEMBRIE NO. 2
DEPARTMENT OF MATHEMATICS

(Received February 13, 1989)