

$D(-1)$ -tuples in the ring $\mathbb{Z}[\sqrt{-k}]$ with $k > 0$

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Abstract. Let n be a non-zero integer and R a commutative ring. A $D(n)$ - m -tuple in R is a set of m non-zero elements in R such that the product of any two distinct elements plus n is a perfect square in R . In this paper, we prove that there does not exist a $D(-1)$ -quadruple $\{a, b, c, d\}$ in the ring $\mathbb{Z}[\sqrt{-k}]$, $k \geq 2$ with positive integers $a < b \leq 8a - 3$ and negative integers c and d . By using that result, we were able to prove that such a $D(-1)$ -pair $\{a, b\}$ cannot be extended to a $D(-1)$ -quintuple $\{a, b, c, d, e\}$ in $\mathbb{Z}[\sqrt{-k}]$ with integers c, d and e . Moreover, we apply the obtained result to the $D(-1)$ -pair $\{p^i, q^j\}$ with arbitrary different primes p, q and positive integers i, j .

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