Relative cohomology dimensions of complexes based on degreewise cotorsion pairs

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Abstract. Let R be an associative ring with identity, and $(\mathcal{A}, \mathcal{B})$ a hereditary cotorsion pair generated by a set in R-Mod. Then $(\mathrm{dw}\, \widetilde{\mathcal{A}}, (\mathrm{dw}\, \widetilde{\mathcal{A}})^{\perp})$ is a complete and hereditary cotorsion pair (we call it the degreewise cotorsion pair) in the category of R-complexes, where $\mathrm{dw}\, \widetilde{\mathcal{A}}$ denotes the class of all complexes X with components $X_n \in \mathcal{A}$ for all $n \in \mathbb{Z}$. For any complexes X and Y and any $n \in \mathbb{Z}$, we define the relative cohomology groups $\mathrm{Ext}^n_{\mathrm{dw}\, \widetilde{\mathcal{A}}}(X, Y)$ based on the degreewise cotorsion pair and investigate the vanishing of the relative cohomology groups. Specifically, we introduce the relative cohomology dimension of X related to $\mathrm{dw}\, \widetilde{\mathcal{A}}$-precovers, and then show that such a dimension of X is equal to the least integer n for which $\mathrm{Ext}^i_{\mathrm{dw}\, \widetilde{\mathcal{A}}}(X, Y) = 0$ for all $i > n$ and all R-modules $Y \in \mathcal{B}$, which recovers the result on relative cohomology dimensions (defined by Liu) of complexes related to Gorenstein projective precovers.

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