Publ. Math. Debrecen
101/1-2 (2022), 147–168
DOI: 10.5486/PMD.2022.9213

On exponential Diophantine equations concerning Pythagorean triples

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Abstract. In 1956, Jeśmanowicz conjectured that the equation $(u^2-v^2)^x+(2uv)^y=(u^2+v^2)^z$ has only the positive integer solution $(x,y,z)=(2,2,2)$, where u and v are positive integers with $u>v$, $\gcd(u,v)=1$ and $u\not\equiv v \mod 2$. Related to Jeśmanowicz' Conjecture, we propose the conjecture that the equation $x^2+(2uv)^m=(u^2+v^2)^n$ has exactly two positive integer solutions $(x,m,n)=(u-v,1,1)$, $(u^2-v^2,2,2)$ except for the cases $(u,v)=(244,231)$ and $3u^2-8uv+3v^2=-1$. We show that this conjecture is true for several cases. The proof is based on the deep results concerning (i) Generalized Lebesgue–Nagell equations, (ii) Generalized Fermat's equations, (iii) Primitive divisors of Lucas numbers, (iv) Linear forms in two logarithms.

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Mathematics Subject Classification: 11D61, 11D41, 11J86.
Key words and phrases: exponential Diophantine equations, generalized Lebesgue–Nagell equations, generalized Fermat's equations, linear forms in two logarithms.