

## Intrinsic metrics under conformal and quasiregular mappings

By OONA RAINIO (Turku)

**Abstract.** The distortion of six different intrinsic metrics and quasi-metrics under conformal and quasiregular mappings is studied in a few simple domains  $G \subsetneq \mathbb{R}^n$ . The already known inequalities between the hyperbolic metric and these intrinsic metrics for points  $x, y$  in the unit ball  $\mathbb{B}^n$  are improved by limiting the absolute values of the points  $x, y$ , and the new results are then used to study the conformal distortion of the intrinsic metrics. For the triangular ratio metric between two points  $x, y \in \mathbb{B}^n$ , the conformal distortion is bounded in terms of the hyperbolic midpoint and the hyperbolic distance of  $x, y$ . Furthermore, quasiregular and quasiconformal mappings are studied, and new sharp versions of the Schwarz lemma are introduced.

OONA RAINIO  
DEPARTMENT OF MATHEMATICS  
AND STATISTICS  
UNIVERSITY OF TURKU  
FI-20014 TURKU  
FINLAND

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