

A lower bound for restricted exponential sums

By MIKHAIL KOMAROV (Vladimir)

Abstract. Let $M(f; r) = \max_{|z|=r} |f(z)|$. Define \mathcal{F}_n , \mathcal{G}_n to be the classes of exponential sums of the form $\sum_{k=1}^n \lambda_k e^{\lambda_k z}$ and $\sum_{k=1}^n e^{\lambda_k z}$, respectively, with $|\lambda_1| = \dots = |\lambda_n| = 1$. For $r \in (0, \frac{1}{2})$, we prove that $\inf_{f \in \mathcal{F}_n} M(f; r) \asymp nr^{n-1}/(n-1)!$, and establish the Turán–Govil type bound $\inf_{g \in \mathcal{G}_n} M(g'; r)/M(g; r) \asymp r^{n-1}/(n-1)!$. Approximations of entire functions of exponential type $\sigma \leq 1$ on compact sets $K \subset \mathbb{C}$ by sums $f_n \in \mathcal{F}_n$, as well as representations of harmonics of a trigonometric polynomial $T_n(t)$ in the form of sums of its translations, $T_n(t-t_k)$, are also considered. In particular, we obtain a new Fejér type estimate for the leading harmonic $\tau_{2n}(t)$ of nonnegative polynomials $T_{2n}(t)$ of even degree $2n$.

MIKHAIL KOMAROV
DEPARTMENT OF FUNCTIONAL ANALYSIS
AND ITS APPLICATIONS
VLADIMIR STATE UNIVERSITY
600000 VLADIMIR
GOR'KOGO STR. 87
RUSSIA

Mathematics Subject Classification: 41A17, 42A05, 30E10.

Key words and phrases: exponential sum, Turán's inequality, approximation, trigonometric polynomial, Fejér's estimate.