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Split extension in Moufang loops

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Abstract. In this paper, we prove the following:

1. Let G be a Moufang loop of order $p^\alpha m$, $(p, m) = 1$, $(p-1, p^\alpha m) = 1$ and p is a prime. Suppose G has an element of order p^α. Then $G = P \rtimes K$, a split extension of a normal subloop K of order m with a subloop P order p^α.

2. Let G be a Moufang loop of odd order $p^2 m$, $(p, m) = 1$, and p is the smallest prime dividing $|G|$. Then a similar result holds as in (1) with $\alpha = 2$.

I. Introduction

Let G be a group of order $p^\alpha m$, $(p, m) = 1$, $(p-1, p^\alpha m) = 1$ and suppose G has an element x of order p^α. Then $G = \langle x \rangle \rtimes K$, a split extension of a normal subgroup K of order m with the subgroup $\langle x \rangle$ [4]. To prove an analogous result on a Moufang loop G, we need a normal subloop K so that we can use induction on G/K. In other words, G has to be nonsimple. By LIEBECK [8], every simple nonassociative Moufang loop is isomorphic to one of the Paige's loop $M^*(q)$. Considering the orders of elements in each of the conjugate classes of $M^*(q)$, as examined by BANNAI and SONG [2], we find that a Moufang loop G with the given properties stated above cannot be simple. Then G has a normal subloop K, and so induction will be possible.

Let G be a group of order, $p^2 m$, $(p, m) = 1$, and p is the smallest prime dividing $|G|$. Then $G = P \rtimes K$, a split extension of a normal subgroup K of order m with the subgroup P of order p^2 [11]. We prove

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also an analogous result on a Moufang loop G by using repeatedly several theorems of GLAUBERMAN [5].

There exist nonassociative Moufang loops of order pq^3 with $q = 1 \pmod{p}$ [10]. Moufang loops of odd order p^2q^3 can be similarly constructed. Hence our two splitting theorems are very useful in studying the structure of such finite Moufang loops.

II. Definitions

1. A binary system $\langle G, \cdot \rangle$, in which specification of any two of the elements x, y, z in the equation $x \cdot y = z$ uniquely determines the third element, is called a quasigroup. If it further contains an identity element, then it is called a loop. Clearly, a group is a loop. But there are loops which are not associative.

2. A loop $\langle G, \cdot \rangle$ is a Moufang loop if $xy \cdot zx = (x \cdot yz)x$ for all x, y, z in G. From now on, G is defined as a finite Moufang loop.

3. Define $zR(x,y) = (zx \cdot y)(xy)^{-1}$,
$zL(x,y) = (yx)^{-1}(y \cdot xz)$ and
$zT(x) = x^{-1} \cdot zx$.

$I(G) = \langle R(x,y), L(x,y), T(x) \mid x, y \in G \rangle$ is called the inner mapping group of G.

4. Let x and y be elements of G. x and y are conjugate if there exists $\theta \in I(G)$ such that $x\theta = y$.

5. Let H be a subloop of G and π a set of primes.

(i) H is a normal subloop of G, $(H \triangleleft G)$, if $H\theta = H$ for all $\theta \in I(G)$ where $H\theta = \{h\theta \mid h \in H\}$.

(ii) H is a π-loop if the order of every element of H is a π-number. (A positive integer n is a π-number if every prime divisor of n lies in π).

(iii) H is a Hall π-subloop of G if $|H|$ is the largest π-number dividing $|L|$.

6. G_a, the associator subloop of G, is the subloop generated by all the associators (x,y,z) where $(x,y,z) = (x \cdot yz)^{-1} \cdot (xy \cdot z)$. Write $G_a = (G,G,G)$ also.

7. G_c, the commutator subloop of G, is the subloop generated by all the commutators $[x, y]$ where $[x, y] = (yx)^{-1} \cdot (xy)$.

8. $N = N(L)$, the nucleus of L, is the subloops generated by all n in L where $(n, x, y) = (x, n, y) = (x, y, n) = 1$ for all x, y in L.

9. $Z = Z(L)$, the centre of L, is the subloop generated by all z in N such that $[z, x] = 1$ for all x in L.

III. Known results with Mounfang loops

Let G be a finite Moufang loop.

$\boldsymbol{R}_1$. (a) $x \in G \Rightarrow |x| \mid |G|$. [1, p. 92, Thm. 1.2].

(*b*) G is disassociative. [1, p. 117, Moufang's Theorem].

(*c*) $x \in G$ and $\theta \in I(G) \Rightarrow x^n\theta = (x\theta)^n$ for any integer n. [1, p. 120, (4.1) and p. 117, Lemma 3.2].

$\boldsymbol{R}_2$. N and Z are normal subloops of G. [1, p. 114, Thm. 2.1 and p. 60, Lemma 1.1]. In fact N and Z are associative by their definitions.

$\boldsymbol{R}_3$. Let H be a normal subloop of G such that $H \subset N$. Then

(*a*) $G/C_G(H) < \operatorname{Aut} H$ where $C_G(H) = \{g \mid g \in G,\ gh = hg$ for all $h \in H\}$.

(*b*) $C_G(H) \cap H = Z(H)$, the centre of H. If $H = N$, then $G_a \subset C_G(N)$. [7, p. 33, Thm. 3].

$\boldsymbol{R}_4$. G is a 2-loop if and only if $|G| = 2^m$ for some positive integer m. [6, p. 415, Thm.].

$\boldsymbol{R}_5$. Suppose $|G|$ is odd and K is a normal subloop of G.

(*a*) If K is minimal normal in G, then K is an elementary Abelian group and $(K, K, G) = 1$. [5, p. 402, Thm. 7].

(*b*) If $(K, K, G) = 1$ and $(|K|, |G/K|) = 1$, then $K \subset N$. [5, p. 405, Thm. 10].

(*c*) G is solvable. [5, p. 413, Thm. 16].

(*d*) G contains a Hall π-subloop, π a set of primes. [5, p. 409, Thm. 12.]

(*e*) If $H < G$ then $|H| \mid |G|$. [5, p. 359, Thm. 2].

$\boldsymbol{R}_6$. If H is a subloop of G, $x \in G$ and d is the smallest positive integer such that $x^d \in H$, then $|\langle H, x\rangle| \geq |H|d$. [3, p. 5, Lemma 0].

$\boldsymbol{R}_7$. There exist simple nonassociative Moufang loops $M^*(p^n)$ with $|M^*(p^n)| = p^{3n}(p^{4n}-1)/d(p)$ where $d(2) = 1$ and $d(p) = 2$ if p is an odd prime. [9, p. 474, Thm. 4.1].

$\boldsymbol{R}_8$. G is a nonassociative simple Moufang loop $\iff$ G is isomorphic with $M^*(p^n)$ for some prime p. [8, p. 33, Theorem].

$\boldsymbol{R}_9$. The conjugacy classes of $M^*(p^n)$ contain elements whose orders are 1, p, divisors of $p^n - 1$ or divisors of $p^n + 1$. [2, p. 224, Thm. 2.1.1 and p. 227, Thm. 2.1.2].

IV. Moufang loops of order $p^\alpha m$

Lemma 1. *Let G be a simple nonassociative Moufang loop of order $2^\alpha m$, $(2, m) = 1$. Then G has no element of order 2^α.*

Proof. By R_7 and R_8, G is isomorphic to $M^*(q)$ for some q where $q = p^n$ and p is a prime. Let x be any 2-element of $M^*(q)$.

Case 1: $p \geq 3$. Let $q - 1 = 2^{\beta_1}m_1$ and $q + 1 = 2^{\beta_2}m_2$, where m_1 and m_2 are odd. Suppose $\beta = \max\{\beta_1, \beta_2\}$. By R_9, $|x| \mid 2^\beta$.

$$\begin{aligned} |M^*(q)| &= \frac{q^3(q^4-1)}{2} = \frac{q^3(q^2+1)}{2}(q-1)(q+1) \\ &= \frac{q^3(q^2+1)}{2}2^{(\beta_1+\beta_2)}m_1m_2 = 2^\alpha m. \end{aligned}$$

Since q is odd, $2 \mid (q^2+1)$. So $\beta_1 + \beta_2 \leq \alpha$. Thus $|x| \leq 2^\beta < 2^{\beta_1+\beta_2} \leq 2^\alpha$ as $\beta_1 > 0$ and $\beta_2 > 0$.

Case 2: $p = 2$. As $q-1$ and $q+1$ are odd, $2 \nmid (q-1)(q+1)$. So by R_9, $|x| = 2$. Now $|M^*(2^n)| = 2^{3n}(2^{4n}-1) = 2^\alpha m$. So $2^\alpha = 2^{3n} \geq 2^3 > 2 = |x|$. Thus G has no element of order 2^α.

Lemma 2. *Let G be a Moufang loop and M a normal subloop of G. Suppose H is a normal Hall π-subloop of M. Then H is normal in G in each of the following two cases:*

(a) *G is of odd order;*

(b) $|M| = 2^r m$ *where* m *is odd,* $|H| = m$ *and there exists an element of order* 2^r *in* M.

PROOF. Suppose $H \not\lhd G$. Then there exists $\theta \in I(G)$ such that $H\theta \neq H$. Since any inner mapping θ is a permutation of G, $H\theta - H \neq \emptyset$.

Let $h\theta \in H\theta - H$. Since $H \lhd M \lhd G$, $H\theta \subset M\theta = M$. Since H and $\langle h\theta \rangle$ are both subloops of M with $H \lhd M$, clearly $H\langle h\theta \rangle$ is a subloop of M. Now by $R_1(c)$, $(h\theta)^{|h|} = \left(h^{|h|}\right)\theta = 1\theta = 1$. So $|h\theta| \mid |h|$. By $R_1(a)$, $|h| \mid |H|$.

So $|h\theta| \mid |H|$. Also $|H\langle h\theta \rangle| = \frac{|H|\,|\langle h\theta \rangle|}{|H \cap \langle h\theta \rangle|}$.

(a) Suppose G is of odd order. Since $|h\theta| \mid |H|$, $H\langle h\theta \rangle$ is a π-loop in M strictly containing the Hall π-subloop H. So $|H\langle h\theta \rangle| \nmid |M|$. This is a contradicition by $R_5(e)$.

So $H \lhd G$ if G is of odd order.

(b) Suppose $|M| = 2^r m$ where m is odd, $|H| = m$ and there exists an element x of order 2^r in M. Since $|h\theta| \mid |H|$, $|H\langle h\theta \rangle|$ is odd. Also $|H\langle h\theta \rangle| > m$ as $h\theta \notin H$. Now by $R_1(a)$, $x^d \notin H\langle h\theta \rangle$ for each $0 < d < 2^r$. But $x^{2^r} = 1 \in H\langle h\theta \rangle$. Thus $|\langle H\langle h\theta \rangle, x \rangle| \geq |H\langle h\theta \rangle| 2^r$ by $R_6 > m2^r = |M|$.

This is a contradiction as $\langle H\langle h\theta \rangle, x \rangle \subset M$. Hence $H \lhd G$ in this case also.

Lemma 3. *Suppose* G *is a Moufang loop of order* $p^\alpha m$, $(p, m) = 1$; K *is a normal subloop of* G *such that* $|G/K| = p^\beta m_0$, $m_0 \mid m$. *Suppose there exists an element* x *of order* p^α *in* G. *Then* xK *is an element of order* p^β *in* G/K.

PROOF. $(xK)^{p^\alpha} = x^{p^\alpha} K = 1K \Rightarrow |xK| \mid p^\alpha \Rightarrow xK$ is a p-element in $G/K \Rightarrow |xK| \mid p^\beta$ by $R_1(a)$. So $|xK| = p^\gamma$, $\gamma \leq \beta$. Then $(xK)^{p^\gamma} = x^{p^\gamma} K = 1K$ and $x^{p^\gamma} \in K$. Since $|K| = p^{\alpha-\beta} m/m_0$ and x^{p^γ} is a p-element in K, $\left|x^{p^\gamma}\right| \mid p^{\alpha-\beta}$ by $R_1(a)$. Thus $\left(x^{p^\gamma}\right)^{p^{\alpha-\beta}} = 1$ or $x^{p^{\alpha+\gamma-\beta}} = 1$. So $\alpha + \gamma - \beta \geq \alpha$. Then $\gamma \geq \beta$. Hence $\gamma = \beta$. So $|xK| = p^\beta$.

Lemma 4. *Let* G *be a Moufang loop of order* $2^\alpha m$, $(2, m) = 1$. *Suppose* G *has an element* x *of order* 2^α. *Then* $G = \langle x \rangle \rtimes K$, *i.e.,* G *is a split extension of a cyclic group* $\langle x \rangle$ *of order* 2^α *with a normal subloop* K *of order* m.

PROOF. If G is a group, we are through by [4, p. 14, Problem 2.16]. So we assume that G is nonassociative. By Lemma 1, we know that G is

nonsimple. Let K be a maximal normal subloop of G. Let $|G/K| = 2^\beta m_0$, $0 \leq \beta \leq \alpha$, $m_0 \mid m$.

Case 1: $1 < m_0 < m : |G/K| = 2^\beta m_0$.

1(a) : $\beta = 0$. Then $|G/K| = m_0$ and $|K| = 2^\alpha(m/m_0)$. By Lemma 3, $|xK| = 1$. Hence $x \in K$. By induction, there exists a subloop K_0 of order m/m_0 normal in K. By Lemma 2, $K_0 \triangleleft G$. Now $|G/K_0| = 2^\alpha m_0$ and xK_0 is an element of order 2^α in G/K_0 by Lemma 3. By induction, there exists a subloop K_1/K_0 of order m_0 normal in G/K_0. Then $K_1 \triangleleft G$ and $|K_1| = |K_0|m_0 = m$. So $G = \langle x \rangle \rtimes K_1$.

1(b) : $\beta \geq 1$. By Lemma 3, xK is an element of order 2^β in G/K. By induction, there exists a subloop K_1/K of order m_0 normal in G/K. Thus $K_1 \triangleleft G$ and $|K_1| = |K|m_0 > |K|$, contradicting the maximality of K.

Case 2: $m_0 = 1 : |G/K| = 2^\beta$.

2(a) : $\beta = 0$. $|G/K| = 1 \Rightarrow G = K$, a contradiction.

2(b) : $0 < \beta < \alpha$. $|K| = 2^{\alpha-\beta}m$. Since $xK \in G/K$, $(xK)^{2^\beta} = 1K$ and $x^{2^\beta} \in K$. Clearly $\left|x^{2^\beta}\right| = 2^{\alpha-\beta}$. By induction, K has a normal subloop K_0 of order m. Thus $K_0 \triangleleft G$ by Lemma 2(b). So $G = \langle x \rangle \rtimes K_0$.

2(c) : $\beta = \alpha$. $|K| = m$ and $G = \langle x \rangle \rtimes K$.

Case 3: $m_0 = m : |G/K| = 2^\beta m$.

3(a) : $\beta = 0$. $|G/K| = m$. Suppose m is not a prime. Then G/K is solvable by $R_5(c)$. So it has proper normal subloop K_1/K. Then $K_1 \triangleleft G$ and $|K| < |K_1| < |G|$. This contradicts that K is a maximal normal subloop of L. So $m = p$, an odd prime. Now $|G| = 2^\alpha p$. By $R_1(a)$, there exists $w \in G$ sucht that $|w| = p$ as otherwise, G would be a 2-loop, which is impossible by R_4. Now by R_6, $G = \langle x, w \rangle$ is a group by diassociativity, a contradiction.

3(b) : $0 < \beta < \alpha$. By Lemma 3, xK is an element of order 2^β in G/K. By induction, there exists a subloop K_1/K of order m normal in G/K. Then $K_1 \triangleleft G$ and $|K_1| = m|K| > |K|$, a contradiction.

3(c) : $\beta = \alpha$. Then $|K| = 1$, a contradiction since K is a maximal normal subloop of G.

Theorem 1. *Let G be a finite Moufang loop of order $p^\alpha m$, $(p, m) = 1$, $(p-1, p^\alpha m) = 1$. Suppose G has an element x of order p^α. Then $G = \langle x\rangle \rtimes K$, i.e., G is a split extension of a cyclic group $\langle x\rangle$ of order p^α and a normal subloop K of order m.*

Proof. By Lemma 4, we can assume that p is an odd prime. Since $(p-1, p^\alpha m) = 1$, G is of odd order. By $R_5(c)$, G is solvable. Let K be a minimal normal subloop of G. By $R_5(a)$, K is an elementary abelian q-group (where q is a prime).

Case 1: $q = p$. $K < \langle x\rangle$. Otherwise, $K\langle x\rangle$ is a p-subloop of G whose order is bigger than p^α, contradicting $R_5(e)$. As $\langle x\rangle$ is cyclic, K is cyclic. So $K = C_p$ as it is an elementary abelian group.

1(a) : $K \not\leq \langle x\rangle$. Then $\alpha \geq 2$, $|G/K| = p^{\alpha-1}m$ and xK is an element of order $p^{\alpha-1}$ by Lemma 3. By induction, there exists a subloop K_1/K of order m normal in G/K. Then $K_1 \triangleleft G$ and $|K_1| = pm$. Now $x^{p^{\alpha-1}}$ is an element of order p in K_1. By induction, there exists a subloop K_2 of order m normal in K_1. Now K_2 is a normal Hall subloop in K_1 and $K_1 \triangleleft G$ implies that $K_2 \triangleleft G$ by Lemma 2(a). Thus $G = \langle x\rangle \rtimes K_2$.

1(b) : $K = \langle x\rangle = C_p$. Now (K, K, G)=1 by $R_5(a)$ and $(|K|, |G/K|)$=1 $\Rightarrow K \subset N$, the nucleus of G, by $R_5(b)$. By $R_3(a)$, $G/C_G(K) \leq \mathrm{Aut}\, K$. As the order of the group of automorphisms of C_p is $p-1$, $\left|\frac{G}{C_G(K)}\right| \Big| p-1$. As $(p-1, |G|) = (p-1, p^\alpha m) = 1$, $G = C_G(K)$. Thus $K \subset Z$, the centre of G. By $R_5(d)$, there exists a Hall subloop H of order m in G. Then $G = HZ$.

Now $\quad G_a = (G, G, G) = (HZ, HZ, HZ) = (H, H, H) \subset H$; and
$\quad\quad G_c = [G, G] = [HZ, HZ] = [H, H] \subset H$.

Let $h \in H$, $x, y \in G$.

Then $\quad hT(x) = x^{-1}hx = hh^{-1}x^{-1}hx = h[h, x]$ and
$\quad\quad hL(x, y) = hR(x^{-1}, y^{-1}), \qquad$ by [1, p. 124, Lemma 5.4, (5.13)]
$\quad\quad\quad\quad = h(h, y, x)^{-1}, \qquad$ by [1, p. 124, Lemma 5.4, (5.16)].

Since $G_a \subset H$ and $G_c \subset H$, $h\theta \in H$ for all $\theta \in I(G)$. Thus $H \triangleleft G$ and $G = \langle x\rangle \triangleleft H$.

Case 2: $q \neq p$. Let $|K| = q^\gamma$. Then $|G/K| = p^\alpha \frac{m}{q^\gamma}$ where $q^\gamma \mid m$.

2(a) : $m > q^\gamma$. By Lemma 3, xK is an element of order p^α in G/K. By induction, there exists a normal subloop K_1/K of order m/q^γ in G/K. Therefore $K_1 \triangleleft G$ and $|K_1| = \frac{|K|m}{q^\gamma} = m$. Thus $G = \langle x \rangle \rtimes K_1$.

2(b) : $m = q^\gamma$. Then $G = \langle x \rangle \rtimes K$ as required.

Corollary 1. *Let G be a Moufang loop of order $p^\alpha m$, $(p, m) = 1$, $(p-1, p^\alpha m) = 1$ and suppose G has an element of order p^α. Then G is solvable.*

Proof. *Case 1:* $p = 2$. Then by Theorem 1, $G = C_{2^\alpha} \rtimes K$ with $|K| = m$ which is odd. So G/K is isomorphic to C_{2^α} which is solvable. By $R_5(c)$, K is solvable. Thus G is solvable.

Case 2: $p \neq 2$. Then $|G|$ is odd as $(p-1, p^\alpha m) = 1$. Thus G is solvable by $R_5(c)$.

V. Moufang loops of odd order $p^2 m$

Theorem 2. *Let G be a Moufang loop of odd order $p^2 m$, $(p, m) = 1$, p the smallest prime dividing $|G|$. Then there exist subloops M and P in G with $|P| = p^2$, $|M| = m$, $M \triangleleft G$ such that $G = P \rtimes M$.*

Proof. If G is a group, we are through by [10, p. 141, 6.3.16]. By $R_5(c)$, G is solvable. Let K be a minimal normal subloop of G. By $R_5(a)$, K is elementary abelian. Let $|K| = q^\alpha$. Existence of P is guaranteed by $R_5(d)$.

Case 1: $q \neq p$. If $|K| = m$, then $K = M$ and we are through. If $|K| < m$, then $|G/K| = p^2 \left({}^{m}\!/_{q^\alpha}\right)$. By induction, there exists a normal subloop M/K in G/K with $|M/K| = \frac{m}{q^\alpha}$. Then $M \triangleleft G$ and $|M| = \frac{m}{q^\alpha} |K| = m$.

Case 2: $q = p$. Then by $R_5(e)$, $\alpha = 1$ or 2.

2(a): $\alpha = 1$: $|K| = p$. By $R_5(d)$, we can get an element xK of order p in G/K. $|G/K| = pm$. So by Theorem 1, there exists a normal subloop $\widehat{M}/K$ of order m in G/K. Then $\widehat{M} \triangleleft G$ and $|\widehat{M}| = pm$. Similarly by $R_5(d)$ and by Theorem 1, there exists a subloop M of order m normal in $\widehat{M}$. By Lemma 2(a), $M \triangleleft G$.

2(*b*): $\alpha = 2 : |K| = p^2$. By $R_5(a)$ and $R_5(b)$, $K \subset N$. Since K is an elementary abelian group, $K = C_p \times C_p$.

Now by $R_3(a)$, $|G/C_G(K)| \mid |\operatorname{Aut} K| = (p+1)p(p-1)^2$ using [10, p. 141, 6.3.15]. Since $K \subset C_G(K)$, and p is the smallest prime dividing $|G|$, $|G/C_G(K)| \mid (p+1)$. As p is odd and 2 does not divide the order of G, $G = C_G(K)$. Thus $K \subset Z$.

By $R_5(d)$, there exists a subloop M of order m in G. As $G = KM = ZM$, it can be shown in a similar way as before (see the proof of Theorem 1, Case 1(b)) that $M \triangleleft G$.

Corollary 2. *Let G be a Moufang loop of odd order $p_1^{\alpha_1} p_2^{\alpha_2} \dots p_m^{\alpha_m}$ where $p_1 < p_2 < \cdots < p_m$ and $1 \leq \alpha_i \leq 2$. Then there exists a subloop of order $p_m^{\alpha_m}$ normal in G.*

Proof. For $\alpha_1 = 1$, $R_5(d)$ guarantees the existence of an element of order p_1 in G. So by Theorem 1 or Theorem 2, there exists M_1, a normal subloop in G with $|M_1| = p_2^{\alpha_2} \dots p_m^{\alpha_m}$. Again there exists a subloop M_2 of order $p_3^{\alpha_3} \dots p_m^{\alpha_m}$ normal in M_1. By Lemma 2(a), $M_2 \triangleleft G$. By this process, we get a subloop M_{m-1} of order $p_m^{\alpha_m}$ normal in G.

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