

Analytic Besov space B^p , $0 < p < 1$

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Abstract. For a measurable function f on the unit ball B in C^n , $n \geq 1$, we define $M_1 f(z)$, $z \in B$, to be the mean modulus of f over a hyperbolic ball with center at z and of a fixed radius. The space $L^{1,p}(\tau)$, $0 < p < 1$, where τ is M -invariant measure on B , is defined by the requirement that $M_1 f \in L^p(\tau)$. The analytic Besov space B^p , $0 < p < 1$, can be naturally embedded as a complemented subspace of $L^{1,p}(\tau)$ by a topological embedding $V_{m,s} : B^p \hookrightarrow L^{1,p}(\tau)$. We show that $V_{m,s} \circ P_s$, where P_s is an integral operator whose reproducing kernel is $\gamma_s(1 - |w|^2)^s(1 - \langle z, w \rangle)^{-(n+1+s)}$, is projection on this embedded copy. The embedding is applied to show that for each $0 < p < 1$ the dual space of the Besov space B^p is isomorphic to the Bloch space B^∞ (with equivalent norms) under certain integral pairing.

1. Introduction

Let B be the open unit ball in C^n , $n \geq 1$, and ν the $2n$ -dimensional Lebesgue measure on B normalized so that $\nu(B) = 1$.

For f analytic on B , $f \in H(B)$, and any positive integer m we write $\partial^m f(z) = (\partial^\alpha f(z))_{|\alpha|=m}$ and $|\partial^m f(z)| = \sum_{|\alpha|=m} |\partial^\alpha f(z)|$, where $\partial^\alpha f(z) = \frac{\partial^{|\alpha|} f(z)}{\partial z^\alpha}$, α a multiindex.

Let $0 < p \leq \infty$ and let m be a positive integer, $m > n/p$. We define the analytic Besov space B^p by

$$B^p = \{f \in H(B) : (1 - |z|^2)^m |\partial^m f(z)| \in L^p(\tau)\},$$

where $d\tau(z) = (1 - |z|^2)^{-n-1} d\nu(z)$ is M -invariant measure on B (see [8]). We note that the definition is independent of m .

Mathematics Subject Classification: 32A35.

Key words and phrases: Besov space, projection, duality.

For $z \in B$, $0 < r < 1$, $E_r(z) = \{w \in B : |\varphi_z(w)| < r\}$. As usual, φ_z is the standard automorphism of B taking 0 to z (see [8]). An integration in polar coordinates shows that $\tau(E_r(z)) = r^{2n}(1-r^2)^{-n} =: \tau(r)$, $z \in B$.

For a complex measurable function f on B we define

$$M_\infty f(z) = M_{\infty,r} f(z) = \text{ess sup}\{|f(w)| : w \in E_r(z)\}$$

and

$$M_p f(z) = M_{p,r} f(z) = \left(\frac{1}{\tau(r)} \int_{E_r(z)} |f(w)|^p d\tau(w) \right)^{1/p}, \quad 0 < p < \infty.$$

For $0 < p, q \leq \infty$, we define $L_r^{p,q}(\tau)$ to be the space of all measurable functions f on B for which

$$\|f\|_{L_r^{p,q}(\tau)} = \|M_{p,r} f\|_{L^q(\tau)} < \infty.$$

Since the definition is independent of r , $0 < r < 1$, we will write $L^{p,q}(\tau)$ instead of $L_r^{p,q}(\tau)$ (see [3] and [11]).

Throughout this paper we assume s is a real number satisfying $s > -1$. Let γ_s be a positive normalizing constant such that the measure $d\nu_s(z) = \gamma_s(1-|z|^2)^s d\nu(z)$ has total mass 1 on B .

Following FORELLI and RUDIN ([4]) we let

$$P_s f(z) = \gamma_s \int_B \frac{(1-|w|^2)^s f(w) d\nu(w)}{(1-\langle z, w \rangle)^{n+1+s}}, \quad z \in B, \quad f \in L^1(\nu_s).$$

We now introduce some differential operators that are of great importance in the rest of the paper.

Let $s > -1$ and $m \geq 0$. We define a linear operator R_s^m on $L^1(\nu_s)$ by

$$R_s^m f(z) = \gamma_s \int_B \frac{(1-|w|^2)^s f(w) d\nu(w)}{(1-\langle z, w \rangle)^{n+1+s+m}}.$$

If $1 \leq p \leq \infty$ and $s > -1$ then $B^p = P_s L^p(\tau)$ (see [7], [10]). In this note we show that the analytic Besov space B^p , $0 < p < 1$, can be naturally embedded as a complemented subspace of $L^{1,p}(\tau)$ by a topological embedding $V_{m,s} : B^p \rightarrow L^{1,p}(\tau)$ defined by $V_{m,s} f(z) = (1-|z|^2)^m R_s^m f(z)$,

where $mp > n$ and $s > -1$. We show that $V_{m,s} \circ P_s$ is projection on this embedded copy and $B^p = P_s L^{1,p}(\tau)$.

Our work was motivated by the paper [6] where the corresponding problem for the Bergman space was considered.

Further, we show that for each $0 < p < 1$ the dual space of the Besov space B^p is isomorphic to the Bloch space B^∞ (with equivalent norms) under certain integral pairing.

2. Analytic Besov space B^p , $0 < p < 1$

First, we give a characterization of the Besov space B^p in terms of the differential operators R_s^m .

Lemma 2.1. *Let $0 < p < \infty$, $s > -1$ and let m be a positive integer, $m > n/p$. If $f \in H(B)$ then the following are equivalent:*

- (i) $f \in B^p$,
- (ii) $\int_B (1 - |z|^2)^{mp} |R^m f(z)|^p d\tau(z) < \infty$, where R denotes the radial derivative $R = \sum_{j=1}^n z_j \frac{\partial}{\partial z_j}$,
- (iii) $\int_B (1 - |z|^2)^{mp} |R_s^m f(z)|^p d\tau(z) < \infty$.

PROOF. In [2] it is shown that (i) $\iff$ (ii)

The implication (ii) $\implies$ (iii) follows from the following identities:

$$(2.1) \quad \begin{aligned} R_s^0 f &= P_s f = f \text{ and} \\ R_s^m f &= [(n+1+s+m)^{-1}R + I] R_s^{m-1} f, \quad f \in B^p, \end{aligned}$$

(a straightforward calculation, see [7]).

Assume now that (iii) holds. Then using the reproducing property of the operator P_s and Fubini's theorem we obtain

$$(2.2) \quad \begin{aligned} &\int_B \frac{(1 - |w|^2)^{s+m} R_s^m f(w)}{(1 - \langle z, w \rangle)^{n+1+s}} d\nu(w) \\ &= \int_B \frac{(1 - |w|^2)^{s+m}}{(1 - \langle z, w \rangle)^{n+1+s}} \left[\gamma_s \int_B \frac{(1 - |\xi|^2)^s f(\xi)}{(1 - \langle w, \xi \rangle)^{n+1+s+m}} d\nu(\xi) \right] d\nu(w) \end{aligned}$$

$$\begin{aligned}
&= \gamma_s \int_B (1 - |\xi|^2)^s f(\xi) \\
&\quad \times \left[\int_B \frac{(1 - |w|^2)^{s+m} d\nu(w)}{(1 - \langle w, \xi \rangle)^{n+1+s+m} (1 - \langle z, w \rangle)^{n+1+s}} \right] d\nu(\xi) \\
&= \frac{\gamma_s}{\gamma_{s+m}} \int_B \frac{(1 - |\xi|^2)^s f(\xi)}{(1 - \langle z, \xi \rangle)^{n+1+s}} d\nu(\xi) = \frac{1}{\gamma_{s+m}} f(z).
\end{aligned}$$

Taking derivatives inside integral gives

$$|\partial^m f(z)| \leq C \int_B \frac{(1 - |w|^2)^{s+m} |R_s^m f(w)|}{|1 - \langle z, w \rangle|^{n+1+s+m}} d\nu(w).$$

By Theorem 1.1 ([5]), $f \in B^p$.

Here and elsewhere constants are denoted by C which may indicate a different constant from one occurrence to the next.

Remark. Carefully examining the proof of Lemma 2.1 above we actually see that the following are equivalent “norms” on B^p for the appropriate p ’s:

$$\begin{aligned}
(1) \quad & \left(\int_B (1 - |z|^2)^{mp} |\partial^m f(z)|^p d\tau(z) \right)^{1/p} + \sum_{|\alpha| < m} |\partial^\alpha f(0)|, \\
(2) \quad & \left(\int_B (1 - |z|^2)^{mp} |R^m f(z)|^p d\tau(z) \right)^{1/p} + |f(0)|, \\
(3) \quad & \left(\int_B (1 - |z|^2)^{mp} |R_s^m f(z)|^p d\tau(z) \right)^{1/p} + |f(0)|.
\end{aligned}$$

In the sequel by $\|f\|_{B^p}$ we will mean any of the expressions (1), (2) and (3).

A similar argument shows that

$$\|f\|_{B^\infty} \cong |f(0)| + \sup_{z \in B} (1 - |z|^2)^m |R_s^m f(z)|.$$

Theorem 2.2. *Let $0 < p < 1$. Then for any $s > -1$, $P_s : L^{1,p}(\tau) \mapsto B^p$ is a continuous linear map. Moreover if m is an integer, $m > n/p$, $V_{m,s} : B^p \mapsto L^{1,p}(\tau)$ is a topological embedding.*

To prove Theorem 2.2 we need the following two lemmas.

Lemma 2.3. *Let $s > -1$ and let k and m be non-negative integers. If $f \in L^1(\nu_s)$, then $R_{s+k}^m R_s^k f = R_s^{m+k} f$.*

PROOF. Same as the equality (2.2).

The following lemma is proven in [6]. The proof is simple and it will be included for a reader convenience.

Lemma 2.4 [6]. *If $0 < p < 1$, then $L^{1,p}(\tau) \subset L^1(\tau)$ and the inclusion map is continuous.*

PROOF. Let $0 < \delta < \epsilon < 1$, $2\delta = \epsilon(1 + \delta^2)$, and let $f \in L_\epsilon^{1,p}(\tau)$. The invariance of the measure τ and Fubini's theorem shows that

$$(2.3) \quad L^q(\tau) = L^{q,q}(\tau), \quad \text{for any } q, \quad 0 < q < \infty.$$

Thus, we have

$$\begin{aligned} \|f\|_{L^1(\tau)} &\cong \|f\|_{L_\delta^{1,1}(\tau)} \leq C \int_B \left[\int_{E_\delta(z)} |f(\xi)| d\tau(\xi) \right] d\tau(z) \\ &\leq C \operatorname{ess\,sup}_{z \in B} \left[\int_{E_\delta(z)} |f(\xi)| d\tau(\xi) \right]^{1-p} \int_B \left[\int_{E_\delta(z)} |f(\xi)| d\tau(\xi) \right]^p d\tau(z). \end{aligned}$$

It is easy to see that if $w \in E_\delta(z)$ then $E_\delta(w) \subset E_\epsilon(z)$. Thus,

$$\int_B [M_{\infty,\delta}(M_{1,\delta}f)(z)]^p d\tau(z) \leq C \int_B [M_{1,\epsilon}f(z)]^p d\tau(z).$$

On the other hand,

$$\begin{aligned} \operatorname{ess\,sup}_{w \in B} (M_{1,\delta}f(w))^p \tau(\delta) &= \operatorname{ess\,sup}_{w \in B} \int_B \mathcal{X}_{E_\delta(w)}(z) (M_{1,\delta}f(w))^p d\tau(z) \\ &\leq \int_B \operatorname{ess\,sup}_{w \in B} \mathcal{X}_{E_\delta(z)}(w) (M_{1,\delta}f(w))^p d\tau(z) \\ &= \int_B [M_{\infty,\delta}(M_{1,\delta}f)(z)]^p d\tau(z). \end{aligned}$$

Combining the above inequalities we find that $\|f\|_{L^1(\tau)} \leq C\|f\|_{L^{1,p}(\tau)}$.

PROOF of Theorem 2.2. Let $f \in L^{1,p}(\tau)$ and let m be any integer larger than n/p . Using Lemma 2.4 we see that

$$\begin{aligned} |R_s^m f(z)| &\leq C \int_B \frac{(1-|w|^2)^{n+1+s} |f(w)|}{|1-\langle z, w \rangle|^{n+1+s+m}} d\tau(w) \\ &\leq C \left[\int_B \left[\int_{E_\epsilon(w)} \frac{(1-|\xi|^2)^{n+1+s} |f(\xi)|}{|1-\langle z, \xi \rangle|^{n+1+s+m}} d\tau(\xi) \right]^p d\tau(w) \right]^{1/p}, \end{aligned}$$

for some fixed ϵ , $0 < \epsilon < 1$.

Since $1 - |\xi|^2 \cong 1 - |w|^2$ and $|1 - \langle z, \xi \rangle| \cong |1 - \langle z, w \rangle|$, if $\xi \in E_\epsilon(w)$, we have

$$|R_s^m f(z)|^p \leq C \int_B \frac{(1-|w|^2)^{p(n+1+s)}}{|1-\langle z, w \rangle|^{p(n+1+s+m)}} \left[\int_{E_\epsilon(w)} |f(\xi)| d\tau(\xi) \right]^p d\tau(w).$$

Using Lemma 2.3 and (2.1) we obtain

$$\begin{aligned} \|(1-|z|^2)^m R_s^m P_s f\|_{L^p(\tau)} &= \|(1-|z|^2)^m R_s^m R_s^0 f\|_{L^p(\tau)} \\ &= \|(1-|z|^2)^m R_s^m f\|_{L^p(\tau)} \leq C \left[\int_B (1-|z|^2)^{mp} \right. \\ &\quad \times \left. \left[\int_B \frac{(1-|w|^2)^{p(n+1+s)}}{|1-\langle z, w \rangle|^{p(n+1+s+m)}} \left[\int_{E_\epsilon(w)} |f(\xi)| d\tau(\xi) \right]^p d\tau(w) \right] d\tau(z) \right]^{1/p} \\ &= C \left[\int_B (1-|w|^2)^{p(n+1+s)} \left[\int_{E_\epsilon(w)} |f(\xi)| d\tau(\xi) \right]^p \right. \\ &\quad \times \left. \left[\int_B \frac{(1-|z|^2)^{mp-n-1} d\nu(z)}{|1-\langle z, w \rangle|^{p(n+1+s+m)}} \right] d\tau(w) \right]^{1/p}, \end{aligned}$$

by Fubini's theorem.

By standard estimates ([8], p. 17)

$$\int_B \frac{(1-|z|^2)^{mp-n-1} d\nu(z)}{|1-\langle z, w \rangle|^{p(n+1+s+m)}} \leq C(1-|w|^2)^{-p(n+1+s)}.$$

Therefore

$$\|(1 - |z|^2)^m R_s^m P_s f\|_{L^p(\tau)} \leq C \|f\|_{L^{1,p}(\tau)}.$$

Using Lemma 2.4 we find that

$$|P_s f(0)| \leq C \|f\|_{L^1(\nu_s)} \leq C \|f\|_{L^1(\tau)} \leq C \|f\|_{L^{1,p}(\tau)}.$$

Thus, $\|P_s f\|_{B^p} \leq C \|f\|_{L^{1,p}(\tau)}$, by the Remark following Lemma 2.1, and we have proved that $P_s : L^{1,p}(\tau) \mapsto B^p$ is continuous.

Let now $f \in B^p$. By (2.3) $\int_B (1 - |z|^2)^{mp} [M_{p,\epsilon} R_s^m f(z)]^p d\tau(z) \leq C \|f\|_{B^p}^p$, for some ϵ , $0 < \epsilon < 1$. Let $0 < \delta < \epsilon < 1$, $2\delta = \epsilon(1 + \delta^2)$. The function $R_s^m f \in H(B)$ and therefore the function $|R_s^m f(\varphi_w)|^p$, $w \in B$, is subharmonic, whence

$$\begin{aligned} |R_s^m f(w)|^p &= |R_s^m f(\varphi_z(0))|^p \leq \delta^{-2n} \int_{\delta B} |R_s^m f(\varphi_w(z))|^p d\nu(z) \\ &\leq \delta^{-2n} \int_{\delta B} |R_s^m f(\varphi_w(z))|^p d\tau(z) = \delta^{-2n} \int_{E_\delta(w)} |R_s^m f(z)|^p d\tau(z). \end{aligned}$$

From this we find that $M_{1,\delta} R_s^m f(z) \leq M_{\infty,\delta} R_s^m f(z) \leq C M_{p,\epsilon} R_s^m f(z)$, $z \in B$.

Thus, $\|(1 - |z|^2)^m M_{1,\delta} R_s^m f\|_{L^p(\tau)} \leq C \|f\|_{B^p}$.

Since

$$\|(1 - |z|^2)^m R_s^m f\|_{L^{1,p}(\tau)} \cong \|(1 - |z|^2)^m M_{1,\delta} R_s^m f\|_{L^p(\tau)},$$

we see that $\|V_{m,s} f\|_{L^{1,p}(\tau)} \leq C \|f\|_{B^p}$.

Using again the fact that $1 - |z|^2 \cong 1 - |w|^2$, if $z \in E_\epsilon(w)$, we get

$$\begin{aligned} \|V_{m,s} f\|_{L^{1,p}(\tau)} &= \left[\int_B \left[\int_{E_\epsilon(w)} (1 - |z|^2)^m |R_s^m f(z)|^p d\tau(z) \right] d\tau(w) \right]^{1/p} \\ &\geq C \left[\int_B (1 - |w|^2)^{mp} \left[\int_{E_\epsilon(w)} |R_s^m f(z)|^p d\tau(z) \right] d\tau(w) \right]^{1/p} \\ &\geq C \left[\int_B (1 - |w|^2)^{mp} \left[\int_{E_\epsilon(w)} |R_s^m f(z)|^p d\tau(z) \right] d\tau(w) \right]^{1/p} \\ &\geq C \|(1 - |z|^2)^m R_s^m f\|_{L^p(\tau)}. \end{aligned}$$

Here we have used the estimate $M_{p,\epsilon} R_s^m f(w) \leq M_{1,\epsilon} R_s^m f(w)$, $w \in B$.

Using (2.2) and Lemma 2.4 we see that

$$|f(0)| \leq C \|V_{m,s} f\|_{L^{1,p}(\tau)}.$$

Thus, $\|f\|_{B^p} \cong \|V_{m,s} f\|_{L^{1,p}(\tau)}$, i.e. $V_{m,s}$ is a topological embedding.

We note that $V_{m,s} \circ P_s$ is projection from $L^{1,p}(\tau)$ onto $V_{m,s}(B^p)$. See the next section. As a corollary of (2.2) and Lemma 2.4 we have that $B^p = P_s L^{1,p}(\tau)$.

3. Duality

A linear functional λ on B^p , $0 < p < 1$, is said to be bounded if

$$\|\lambda\| = \sup\{|\lambda(f)| : \|f\|_{B^p} \leq 1\} < \infty.$$

The dual space of B^p , denoted $(B^p)^\star$, is then the space of all bounded linear functionals on B^p .

In [12] it is shown that each dual $(L_a^{p,s})^\star$, $0 < p < 1$, $s > -1$, of the weighted Bergman space $L_a^{p,s} = L^p(\nu_s) \cap H(B)$, can be indentified with B^∞ via volume integral pairing

$$\langle f, g \rangle_\beta = \lim_{r \rightarrow 1} \int_B f(rz) \overline{g(z)} d\nu_\beta(z), \quad f \in L_a^{p,s}, \quad g \in B^\infty,$$

where $\beta = \frac{n+1+s}{p} - (n+1)$. In this note we show that $(B^p)^\star$, $0 < p < 1$, can also be indentified with B^∞ , but via a different pairing.

Theorem 3.1. *Let $0 < p < 1$, let m be a positive integer, $mp > n$, and $s = m - n - 1$. The integral pairing*

$$\langle f, g \rangle_\tau = \int_B V_{m,s} f(z) \overline{V_{m,s} g(z)} d\tau(z)$$

induces the following duality: $(B^p)^\star = B^\infty$.

PROOF. First, assume that g is a function in B^∞ . Then

$$\sup_{z \in B} |V_{m,s} g(z)| = \sup_{z \in B} (1 - |z|^2)^m |R_s^m g(z)| \leq C \|g\|_{B^\infty}.$$

We show that g gives rise to a bounded linear functional on B^p under the pairing $\langle \cdot, \cdot \rangle_\tau$. By Theorem 2.2 and Lemma 2.4 if $f \in B^p$ then

$$\|f\|_{B^p} \geq C \|V_{m,s}f\|_{L^{1,p}(\tau)} \geq C \|V_{m,s}f\|_{L^1(\tau)}.$$

Thus if $f \in B^p$, then we have

$$|\langle f, g \rangle|_\tau \leq \sup_{z \in B} |V_{m,s}g(z)| \|V_{m,s}f\|_{L^1(\tau)} \leq C \|f\|_{B^p} \|g\|_{B^\infty}$$

Conversely, assume that λ is a bounded linear functional on B^p ; we show that λ arises from a function in B^∞ . Since $V_{m,s}$ is a topological embedding of B^p into $L^{1,p}(\tau)$, $\lambda \circ V_{m,s}^{-1}$ is a bounded linear functional on the image space of $V_{m,s}$ in $L^{1,p}(\tau)$. Since $L^{1,p}(\tau) \subset L^1(\tau)$, by the Hahn-Banach theorem $\lambda \circ V_{m,s}^{-1}$ extends to a bounded linear functional on $L^1(\tau)$. Thus, there exists a function $\varphi \in L^\infty(\tau)$ such that

$$\lambda \circ V_{m,s}^{-1}(\psi) = \int_B \psi(z) \overline{\varphi(z)} d\tau(z), \quad \psi \in L^1(\tau).$$

When $f \in B^p$, then $V_{m,s}f \in L^{1,p}(\tau) \subset L^1(\tau)$. Therefore

$$\lambda(f) = \int_B V_{m,s}f(z) \overline{\varphi(z)} d\tau(z), \quad f \in B^p.$$

Let $h = P_s(\varphi)$. Then $h \in B^\infty$ and by Lemma 2.3

$$V_{m,s}h(z) = (1 - |z|^2)^m R_s^m(P_s\varphi)(z) = (1 - |z|^2)^m R_s^m\varphi(z) = V_{m,s}\varphi(z).$$

To finish the proof of Theorem 3.1 it remains to show that

$$\langle V_{m,s}f, V_{m,s}\varphi \rangle_\tau = \langle V_{m,s}^2f, \varphi \rangle_\tau \quad \text{and} \quad \gamma_{m+s}\gamma_s^{-1}V_{m,s}^2f = V_{m,s}f.$$

Note that $s = m - n - 1$. This follows easily from Fubini's theorem and the reproducing property of P_s . We leave the details to the interested reader.

Thus,

$$\lambda(f) = \int_B V_{m,s}f(z) \overline{V_{m,s}g(z)} d\tau(z), \quad f \in B^p,$$

where $g = \gamma_{s+m}\gamma_s^{-1}h \in B^\infty$.

The proof of Theorem 3.1 shows that the dual space of B^1 can also be identified with B^∞ under the integral pairing $\langle \cdot, \cdot \rangle_\tau$. See also [13].

Using the term “Mackey topology” (see [9]), we can rephrase Theorem 3.1 as follows.

Corollary 3.2. *Suppose $0 < p < 1$. Then the norm of B^1 induces the Mackey topology on B^p ; and B^1 is the Mackey completion of B^p .*

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(Received February 4, 1997)