

## Naturally reductive homogeneous $(\alpha, \beta)$ spaces

By SHAOXIANG ZHANG (Qingdao), ZAILI YAN (Ningbo)  
and SHAOQIANG DENG (Tianjin)

**Abstract.** In this paper, we study naturally reductive homogeneous Finsler spaces. In the literature, there are two versions of the definition for naturally reductive Finsler spaces. Our first main result shows that the two versions are equivalent. Then we study naturally reductive  $(\alpha, \beta)$ -metrics and give an explicit formula for flag curvature of naturally reductive  $(\alpha, \beta)$ -metrics. Finally, we compute the flag curvature of several important  $(\alpha, \beta)$ -metrics, including Randers, Berwald square, Matsumoto and Kropina metrics.

SHAOXIANG ZHANG  
COLLEGE OF MATHEMATICS  
AND SYSTEMS SCIENCE  
SHANDONG UNIVERSITY OF  
SCIENCE AND TECHNOLOGY  
266590 QINGDAO  
P. R. CHINA

ZAILI YAN  
DEPARTMENT OF MATHEMATICS  
NINGBO UNIVERSITY  
315211 NINGBO  
P. R. CHINA

SHAOQIANG DENG  
SCHOOL OF MATHEMATICAL  
SCIENCES AND LPMC  
NANKAI UNIVERSITY  
300071 TIANJIN  
P. R. CHINA

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