

Congruences for sums of powers of an integer

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Abstract. For coprime integers q and e , let $m(q, e)$ denote the least positive integer t such that there exists a sum of t powers of q which is divisible by e . We prove that $m(q, e) \leq \lceil e/\text{ord}_e(q) \rceil$ where $\text{ord}_e(q)$ denotes the (multiplicative) order of q modulo e . We apply this in order to classify, for any positive integer r , the cases where $m(q, e) \geq \frac{e}{r}$ and $e > r^4 - 2r^2$. In particular, we determine all pairs (q, e) such that $m(q, e) \geq \frac{e}{6}$. We also investigate in more detail the case where e is a prime power.

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