Multiplicative closure of generalized invertible elements in a ring

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Abstract. Let R be a ring. It is shown that the subset $R^{\{1\}}$ of $\{1\}$-invertible elements of R is a multiplicative set if and only if R is an SSP ring. As a quick application, it is proved that the upper triangular matrix ring $\mathrm{T}_n(R)$ $(n \geq 2)$ over any ring R is never SSP. In the case of a ring R with involution, multiplicative property of $R^{\{1,3\}}$, $R^{\{1,4\}}$ and $R^\dagger$ are also characterized. It is shown that $R^{\{1,3\}}$ is a multiplicative set if and only if $R^{\{1,4\}}$ is a multiplicative set, which also implies that $R^\dagger$ is a multiplicative set.

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