On the powerful numbers in $\prod_{x=g}^{f}(x^k \pm h^k)$

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Abstract. When k is odd with at most t distinct prime factors, for $Q = \frac{1}{2}$ and positive integers f, g, h belonging to some specific sequences, results from the literature indicate that there are constants $C(Q,t)$ such that $C = \prod_{x=g}^{f}(x^k + h^k)$ is not a powerful number if $f + h \geq \max\{C(Q,t), \frac{1}{Q}(g + h - 1)\}$. When k is odd, it is proved that C is not a powerful number if $f + h \geq \max\{10^6, \frac{1}{Q}(g + h - 1)\}$ for any f, g, h and any $Q \in [0.5, 0.89963]$. Similar conclusions on $D = \prod_{x=g}^{f}(x^k - h^k)$ are also proved.

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