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Abstract. Let $n \geq 2$ be an integer and $\alpha_1, \dots, \alpha_n$ be non-zero algebraic numbers. Let b_1, \dots, b_n be integers with $b_n \neq 0$, and set $B = \max\{3, |b_1|, \dots, |b_n|\}$. For $j = 1, \dots, n$, set $h^*(\alpha_j) = \max\{h(\alpha_j), 1\}$, where h denotes the (logarithmic) Weil height. Assume that the quantity $\Lambda = b_1 \log \alpha_1 + \dots + b_n \log \alpha_n$ is nonzero. A typical lower bound of $\log |\Lambda|$ given by Baker's theory of linear forms in logarithms takes the shape

$$\log |\Lambda| \geq -c(n, D) h^*(\alpha_1) \cdots h^*(\alpha_n) \log B,$$

where $c(n, D)$ is positive, effectively computable and depends only on n , and on the degree D of the field generated by $\alpha_1, \dots, \alpha_n$. However, in certain special cases and in particular when $|b_n| = 1$, this bound can be improved to

$$\log |\Lambda| - c(n, D) h^*(\alpha_1) \cdots h^*(\alpha_n) \log \frac{B}{h^*(\alpha_n)}.$$

The term $B/h^*(\alpha_n)$ in place of B originates in works of Feldman and Baker and is a key tool for improving, in an effective way, the upper bound for the irrationality exponent of a real algebraic number of degree at least 3 given by Liouville's theorem. We survey various applications of this refinement to exponents of approximation evaluated at algebraic numbers, to the S -part of some integer sequences, and to Diophantine equations. We conclude with some new results on arithmetical properties of convergents to real numbers.

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