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On the denominators of generalized harmonic numbers. II

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Abstract. For three positive integers a, b and n, let $H_{a,b}(n)$ be the sum of the reciprocals of the first n terms of arithmetic progression $\{ak+b : k = 0, 1, \dots\}$, and let $v_{a,b}(n)$ be the denominator of $H_{a,b}(n)$. In this paper, we prove that the set of positive integers n satisfying $\nu_p(v_{a,b}(n)) = \nu_p([b, b+a, \dots, b+(n-1)a])$ has positive logarithmic density, where $[b, b+a, \dots, b+(n-1)a]$ denotes the least common multiple of $b, b+a, \dots, b+(n-1)a$.

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