

Exceptional sets related to the product of consecutive digits in Lüroth expansions

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Abstract. Every real number $x \in (0, 1]$ admits a Lüroth expansion $[d_1(x), d_2(x), \dots]_L$ with $d_n(x) \in \mathbb{N}_{\geq 2}$ being its digits. Let $\{\frac{p_n(x)}{q_n(x)}, n \geq 1\}$ be the sequence of convergents of the Lüroth expansion of x . We study the growth rate of the product of consecutive digits relative to the denominator of the convergent for the Lüroth expansion of an irrational number. More precisely, given a natural number m , we prove that the set

$$E_m(\beta) = \left\{ x \in (0, 1] : \limsup_{n \rightarrow \infty} \frac{\log(d_n(x)d_{n+1}(x) \cdots d_{n+m}(x))}{\log q_n(x)} = \beta \right\}$$

and the set

$$\tilde{E}_m(\beta) = \left\{ x \in (0, 1] : \limsup_{n \rightarrow \infty} \frac{\log(d_n(x)d_{n+1}(x) \cdots d_{n+m}(x))}{\log q_n(x)} \geq \beta \right\}$$

share the same Hausdorff dimension for $\beta \geq 0$. It significantly generalises the existing results on the Hausdorff dimension of $E_1(\beta)$ and $\tilde{E}_1(\beta)$.

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