

## Uniqueness conjecture on simultaneous Pell equations. II

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**Abstract.** Let  $A$  and  $B$  be distinct positive integers. It is known that any positive solution to the simultaneous Pell equations  $x^2 - Ay^2 = 1$  and  $z^2 - By^2 = 1$  gives rise to a positive solution to the simultaneous Pell equations  $x^2 - (m^2 - 1)y^2 = 1$  and  $z^2 - (n^2 - 1)y^2 = 1$  for some distinct integers  $m$  and  $n$  greater than one. In this paper, we prove that the latter equations have only the positive solution  $(x, y, z) = (m, 1, n)$  if  $\{1, b, c\}$  is a Diophantine triple with  $b = m^2 - 1$ ,  $c = n^2 - 1$  and  $c \geq \max\{200b^4, 2b^5\}$ . Moreover, we show that the same conclusion holds if we replace the inequality assumed above by  $b = \sigma p^e + 1$  for some prime  $p$ , a positive integer  $e$  and  $\sigma \in \{1, 2, 4\}$ .

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